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Mariner B Telecommunication System Reliability Study

Man K. Tam

JET PROPULSION LABORATORY
CALIFORNIA INSTITUTE OF TECHNOLOGY
PASADENA, CALIFORNIA

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J. F. Koukol

J. F. Koukol, Chief
Communication System Development Section

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JET PROPULSION LABORATORY
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CONTENTS

CONTENTS (Cont'd)

CONTENTS (Cont'd)

TABLES

FIGURES

FIGURES (Cont'd)

FIGURES (Cont'd)

PREFACE

In September, 1962, Applied Development Corporation of Monterey Park was contracted by the Jet Propulsion Laboratory (Contract No. B3-205569) to perform a reliability study of the *Mariner* B spacecraft communication system. During the course of this study, mathematical expressions describing the functional reliability of the before-mentioned system, based on the criteria of partial successes, were derived. These mathematical expressions and graphical illustrations of the relative reliabilities of various functions are presented in this Memorandum.

The author wishes to acknowledge the able guidance of Richard P. Mathison of the Communication System Development Section of JPL. Thanks are also due to Donall G. Bourke of the same section for his helpful criticisms and suggestions throughout this study. Contributions to this study by the entire staff of the Communication System Development and the R.F. Systems Development Sections of JPL are greatly appreciated.

ABSTRACT

15980

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Functional reliability equations of the *Mariner* B spacecraft telecommunication system are derived by using event algebra and a reliability model constructed under considerations of partial successes. These equations are enumerated and are given in terms of other dependents within the spacecraft. Numerical values for the functional reliabilities are obtained on a parts-count basis. Three-dimensional graphical displays are constructed for each of the functional equations.

Reliability improvement in general is considered from the standpoint of functional blocks as well as on the basis of individual circuitry. A comprehensive comparison between the results of the *Mariner* A and the *Mariner* B reliability studies is presented. Through considerations of the results of these studies, some comments and recommendations are also given.

AUTHOR

I. DERIVATION OF RELIABILITY EQUATIONS OF GIVEN SPACECRAFT COMMUNICATION FUNCTIONS

A. Introduction

The reliability of a system may be defined as the probability of success of the system in performing a given mission over a specific period of time. Consequently, the reliability analysis is confined to the basic concepts of probability. In recent years, there has been a strong tendency to apply "set theory" in solving probability problems. One considers a sample space that is a set whose points are called sample points. Each sample point $\mathcal{E}_j$ represents a particular event for which the associated probability Pr $[\mathcal{E}_j]$ has the following properties:

1. For each $\mathcal{E}_j$, $0 \leq \Pr[\mathcal{E}_j] \leq 1$.

2. For all $\mathcal{E}_j$ within the sample space, $\sum_{j=1}^{n} \Pr[\mathcal{E}_j] = 1$.

A combination of set theory applications and probability theorems constitutes a unique tool for the derivation of the reliability equations of complex systems. The function-success event equation may be reduced to a convenient form by means of set theory algebra; the probability equation for the same function may be obtained by applying probability theorems to the event equation. The following is a summary of the necessary mathematical background.

1. Laws Applied to the Algebra of Sets

The following laws are applicable to the algebra of sets and will be used without reserve in the derivation of reliability equations:

(1) *Idempotent.*

$$\mathcal{E}_1 + \mathcal{E}_1 = \mathcal{E}_1 \qquad\qquad \mathcal{E}_1 \cdot \mathcal{E}_1 = \mathcal{E}_1$$

(2) *Commutative.*

$$\mathcal{E}_1 + \mathcal{E}_2 = \mathcal{E}_2 + \mathcal{E}_1 \qquad\qquad \mathcal{E}_1 \cdot \mathcal{E}_2 = \mathcal{E}_2 \cdot \mathcal{E}_1$$

(3) *Associative.*

$$(\mathcal{E}_1 + \mathcal{E}_2) + \mathcal{E}_3 = \mathcal{E}_1 + (\mathcal{E}_2 + \mathcal{E}_3) \qquad\qquad (\mathcal{E}_1 \cdot \mathcal{E}_2) \cdot \mathcal{E}_3 = \mathcal{E}_1 \cdot (\mathcal{E}_2 \cdot \mathcal{E}_3)$$

(4) *Distributive.*

$$\mathcal{E}_1 + (\mathcal{E}_2 \cdot \mathcal{E}_3) = (\mathcal{E}_1 + \mathcal{E}_2) \cdot (\mathcal{E}_1 + \mathcal{E}_3) \qquad\qquad \mathcal{E}_1 \cdot (\mathcal{E}_2 + \mathcal{E}_3) = \mathcal{E}_1 \cdot \mathcal{E}_2 + \mathcal{E}_1 \cdot \mathcal{E}_3$$

(5) *Dualization.*

$$\overline{\mathcal{E}_1 + \mathcal{E}_2} = \bar{\mathcal{E}}_1 \cdot \bar{\mathcal{E}}_2$$

(6) *Complementary.*

$$\mathcal{E}_1 \cdot \bar{\mathcal{E}}_1 = \mathcal{N}$$

$$\mathcal{E}_1 + \bar{\mathcal{E}}_1 = \mathcal{I}$$

$$\bar{\mathcal{E}}_1 = \mathcal{I} - \mathcal{E}_1$$

where

$\mathcal{I}$ = the universal set under discourse

$\mathcal{N}$ = the null set

2. Operations of the Algebra of Sets

a. Set Inclusion.

$\mathcal{E}_1 \subset \mathcal{E}_2$ means $\mathcal{E}_1$ is contained in $\mathcal{E}_2$.

$\mathcal{E}_1 \supset \mathcal{E}_2$ means $\mathcal{E}_1$ contains $\mathcal{E}_2$.

For all sets $\mathcal{E}_j$, $\mathcal{I} \supset \mathcal{E}_j \supset \mathcal{N}$.

If $\mathcal{E}_1 \supset \mathcal{E}_2$ and $\mathcal{E}_2 \supset \mathcal{E}_1$, then $\mathcal{E}_1 = \mathcal{E}_2$.

b. Summation of Sets.

$$\mathcal{E}_3 = \mathcal{E}_1 + \mathcal{E}_2$$

The summation of the sets $\mathcal{E}_1$ and $\mathcal{E}_2$ forms a new set $\mathcal{E}_3$ that contains all points within the sets $\mathcal{E}_1$ and $\mathcal{E}_2$.

c. Product of Sets.

$$\mathcal{E}_4 = \mathcal{E}_1 \cdot \mathcal{E}_2$$

The product of the sets $\mathcal{E}_1$ and $\mathcal{E}_2$ forms a new set $\mathcal{E}_4$ that contains all points common to the sets $\mathcal{E}_1$ and $\mathcal{E}_2$.

d. Subtraction of Sets.

$$\mathcal{E}_5 = \mathcal{E}_1 - \mathcal{E}_2$$

The difference of the sets $\mathcal{E}_1$ and $\mathcal{E}_2$ forms a new set $\mathcal{E}_5$ that contains all points of set $\mathcal{E}_1$, excluding those of set $\mathcal{E}_2$. If the set $\mathcal{E}_1$ does not contain any points of the set $\mathcal{E}_2$, the subtraction operation is meaningless.

3. Basic Theorems of Probability

a. Addition Theorem. The probability of occurrence of the union of two events $\mathcal{E}_1$ and $\mathcal{E}_2$ is

$$\Pr[\mathcal{E}_1 + \mathcal{E}_2] = \Pr[\mathcal{E}_1] + \Pr[\mathcal{E}_2] - \Pr[\mathcal{E}_1 \cdot \mathcal{E}_2] \tag{1}$$

where $\Pr[\mathcal{E}_1 \cdot \mathcal{E}_2]$ is the joint occurrence of events $\mathcal{E}_1$ and $\mathcal{E}_2$.

b. Multiplication Theorem. The joint probability of two events is equal to the probability of one of the events times the conditional probability that the second event will occur, given that the first event has occurred:

$$\Pr[\mathcal{E}_1 \cdot \mathcal{E}_2] = \Pr[\mathcal{E}_1] \cdot \Pr[\mathcal{E}_2 | \mathcal{E}_1] \tag{2}$$

If the two events are independent,

$$\Pr[\mathcal{E}_1 \cdot \mathcal{E}_2] = \Pr[\mathcal{E}_1] \cdot \Pr[\mathcal{E}_2] \tag{3}$$

c. Mutual Exclusiveness. Two events $\mathcal{E}_1$ and $\mathcal{E}_2$ are mutually exclusive if

$$\Pr[\mathcal{E}_1 \cdot \mathcal{E}_2] = 0 \tag{4}$$

It follows that two events can be both independent and mutually exclusive if and only if either $\Pr[\mathcal{E}_1]$ or $\Pr[\mathcal{E}_2]$ is zero.

d. Bayes' Theorem. Bayes' theorem may be written in a simplified form that offers great use in reliability analysis:

$$\Pr [\mathcal{A}] = \sum_{j=1}^{n} \Pr [\mathcal{A} | \mathcal{E}_j] \cdot \Pr [\mathcal{E}_j] \tag{5}$$

where

$$\sum_{j=1}^{n} \Pr [\mathcal{E}_j] = 1$$

and the $\mathcal{E}_j$'s are mutually exclusive.

For a given function to be analyzed, a reliability diagram is first constructed. This reliability diagram consists of all the events involved and may be considered as the sample space of this function. To simplify the analysis, a systematic approach is proposed, and a set of rules for the reduction of the system to individual blocks is generated. A detailed description of these rules will be given in Section I-B.

In the *Mariner B* spacecraft communication system, reliability equations for the following functions will be developed:

1. Command function
2. Doppler-tracking function
3. Range-tracking function
4. Telemetry function

The derivations are based on the currently available documents and drawings pertaining to these functions. Every attempt has been made to accomodate any conceivable change in system mechanization without significant alteration of the reliability equations.

B. Reliability Diagrams

In an attempt to derive the reliability equation of a given function in a complex system, effort can be reduced if a reliability diagram for that function is first prepared. A reliability diagram, unlike its functional schematic counterpart, does not necessarily indicate the actual interconnections among functional blocks or the actual signal flow path, but it does show the exact relationship of individual block failures to the functioning of the system. To illustrate this statement, consider a conventional amplifier with an external power supply. In the functional block diagram shown in Fig. 1(a), the power supply block is connected to the amplifier, with a control line between the two blocks. However, in the reliability diagram shown in Fig. 1(b), the two blocks are connected in series because of the success dependency of these blocks. This series combination of blocks may be identified as a branch, and the failure of any block along a branch causes the failure of the entire branch. Symbolically, a branch may be represented by a single block. In general, for a complex system, the reliability diagram is constructed with many branches interconnected in a highly complicated manner. To prepare the reliability diagram, the prime objective is to reduce a complicated connection to the simpler series construction without losing the significance of the reliability measure of the original diagram.

For future reference, a collection of frequently seen basic configurations is presented. For each configuration, the reliability diagram and its associated equations are given. Emphasis is placed upon some of the particular switching configurations that play important roles in the reliability analyses of partial successes.

1. Series Configuration

Consider a system (Fig. 2) consisting of n functional blocks, where the failure of any one of these blocks causes the system function to fail. Let $\mathcal{E}_j$ be the success event for block j, and let $\mathcal{S}$ be the success event for the system function. Then the success event of the system function is given by

$$\mathcal{S} = \mathcal{E}_1 \cdot \mathcal{E}_2 \cdot \dots \cdot \mathcal{E}_n \tag{6}$$

If all $\mathcal{E}_j$'s are independent, then

$$\begin{aligned}\Pr [\mathcal{S}] &= \Pr [\mathcal{E}_1] \cdot \Pr [\mathcal{E}_2] \cdot \dots \cdot \Pr [\mathcal{E}_n] \\ &= \prod_{j=1}^{n} \Pr [\mathcal{E}_j]\end{aligned} \tag{7}$$

Otherwise, set algebra must be applied to Eq. (7) until the independence of each function is assured.

2. Parallel Configuration

Consider a system consisting of n components (Fig. 3). If any one of these components operates properly, the system function is considered fulfilled. The success events of the individual components are denoted by $\mathcal{E}_j$, and the success event of the system function is denoted by $\mathcal{S}$. Hence

$$\mathcal{S} = \mathcal{E}_1 + \mathcal{E}_2 + \cdots + \mathcal{E}_n \tag{8}$$

Several methods may be used to obtain the probability of success of the event $\mathcal{S}$. In general, the probability of success for n parallel blocks may be obtained by applying the addition theorem (Eq. 1):

$$\begin{aligned} \Pr[\mathcal{S}] = {} & \sum_{j=1}^{n} \Pr[\mathcal{E}_j] - \sum_{\substack{\text{All combinations of} \\ \text{i and j, } i \neq j}} \Pr[\mathcal{E}_i \cdot \mathcal{E}_j] \\ & + \sum_{\substack{\text{All combinations of} \\ \text{i, j, and k, } i \neq j \neq k}} \Pr[\mathcal{E}_i \cdot \mathcal{E}_j \cdot \mathcal{E}_k] \\ & \cdots + (-1)^{n-1} \Pr[\mathcal{E}_1 \cdot \mathcal{E}_2 \cdot \cdots \cdot \mathcal{E}_n] \end{aligned} \tag{9}$$

If all $\mathcal{E}_j$'s are mutually exclusive,

$$\Pr[\mathcal{S}] = \sum_{j=1}^{n} \Pr[\mathcal{E}_j]$$

An alternate method for obtaining $\Pr[\mathcal{S}]$ is to utilize the dualization theorem and the subtraction operation of set theory. The complement of the event $\mathcal{S}$ is given by

$$\overline{\mathcal{S}} = \overline{\mathcal{E}}_1 \cdot \overline{\mathcal{E}}_2 \cdot \cdots \cdot \overline{\mathcal{E}}_n \tag{10}$$

and

$$\mathcal{S} = 1 - \overline{\mathcal{S}}$$

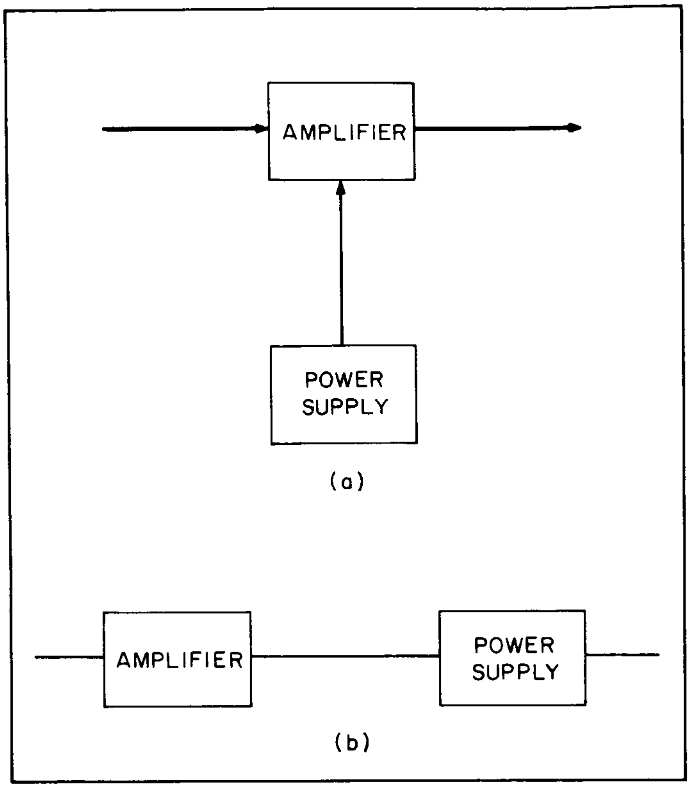
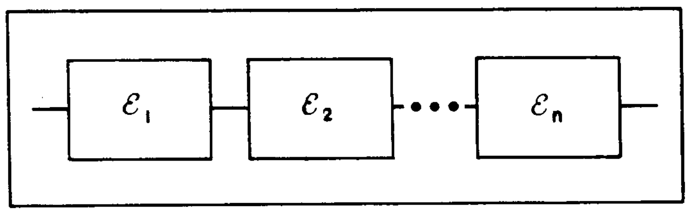


Fig. 1. Functional and reliability diagrams for a simple amplifying system

a. Functional block diagram
b. Reliability diagram

Fig. 2. Series combination of n functional blocks



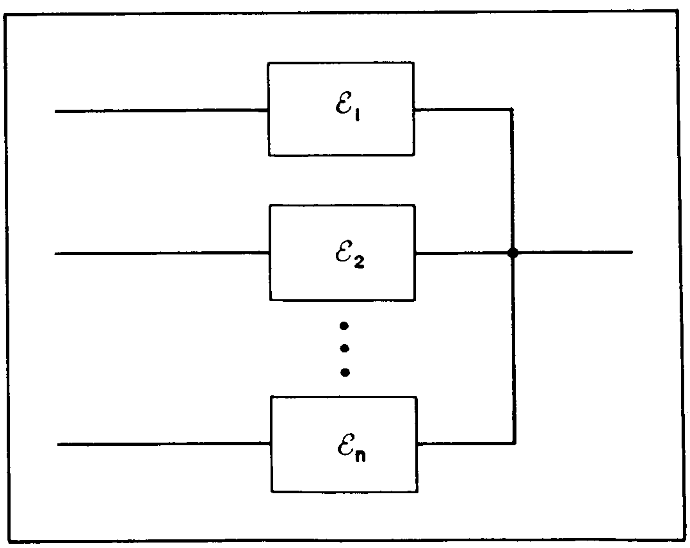


Fig. 3. Parallel combination of n functional blocks

Therefore,

$$\Pr[\mathcal{S}] = 1 - \Pr[\overline{\mathcal{S}}] = 1 - \prod_{j=1}^{n} \Pr[\overline{\mathcal{E}}_j] \tag{11}$$

provided that all $\overline{\mathcal{E}}_j$'s are independent. Note that if the $\mathcal{E}_j$'s are mutually exclusive, the alternate method cannot be directly applied since the $\overline{\mathcal{E}}_j$'s are not independent.

3. Stand-by Configuration

Let n identical units be arranged in such a way that only one unit will be operating at a given time, with no power applied to the other units (Fig. 4). As soon as the operating unit fails, the second unit is automatically switched in and performs the function of the failed unit. In the stand-by arrangement, operating time becomes a function of the success event, and there is no proper description available from set theory. Consequently, this model will be used to obtain only the probability of success of an independent block with this configuration.

To evaluate the probability of success of a stand-by configuration with n units, (neglecting the detecting and switching circuits), one may make use of the Poisson distribution. It is well known that

$$\Pr[\mathcal{S}] + \Pr[\overline{\mathcal{S}}] = 1$$

$$\Pr[\mathcal{S}] + \Pr[\overline{\mathcal{S}}] = e^{-f} \cdot e^{f} \tag{12}$$

where

$\Pr[\mathcal{S}]$ = probability of system success

$\Pr[\overline{\mathcal{S}}]$ = probability of system failure

f = failure = t/τ

t = total system operating time

τ = mean time between failures

Expanding the right-hand side of Eq. (12),

$$\Pr[\mathcal{S}] + \Pr[\bar{\mathcal{S}}] = e^{-f}\left[1 + f + \frac{f^2}{2!} + \cdots\right] = e^{-f} + f e^{-f} + \frac{f^2}{2!} e^{-f} + \cdots \tag{13}$$

The probability of success of n units operating in the stand-by manner is

$$\Pr[\mathcal{S}] = \left[\sum_{i=0}^{n-1} \frac{f^i}{i!}\right] e^{-f} \tag{14}$$

4. Feedback Control Configuration

Quite frequently, a system is constructed as shown in Fig. 5 whereby the system is considered successful if the input signal is properly transmitted from the input to the output. The output of this system may be a number of control signals, and one or more of these signals are used through some circuitry $\mathcal{F}$ to regulate the functional block $\mathcal{E}_1$. The success event of this configuration is given by

$$\mathcal{S} = \mathcal{E}_1 \cdot \mathcal{E}_2 \cdot \text{(control success event)} \tag{15}$$

since the failure of any one of the above functions causes the system operation to fail. The control success event is based on the signal transfer and the success of the feedback mechanism $\mathcal{F}$:

$$\mathcal{C} = \mathcal{E}_1 \cdot \mathcal{E}_2 \cdot \mathcal{F} \tag{16}$$

Combining Eq. (15) and (16) yields

$$\mathcal{S} = \mathcal{E}_1 \cdot \mathcal{E}_2 \cdot (\mathcal{E}_1 \cdot \mathcal{E}_2 \cdot \mathcal{F}) = \mathcal{E}_1 \cdot \mathcal{E}_2 \cdot \mathcal{F} \tag{17}$$

Equation (17) suggests that the system reliability diagram can be reduced to a series configuration, as given in Fig. 6.

The probability of success is

$$\Pr[\mathcal{S}] = \Pr[\mathcal{E}_1 \cdot \mathcal{E}_2 \cdot \mathcal{F}] \tag{18}$$

5. Simple Controlled Switching Configuration

Figure 7 illustrates a simple form of controlled switch in which block $\mathcal{K}$ is a single-pole single-throw switch and block $\mathcal{C}$ is an external control function. The following definitions are created:

$\mathcal{C}$ = success event of the control function

$\overline{\mathcal{C}}$ = failure event of the control function

$\mathcal{K}$ = success event of the switch (The failure of the switch is limited to the contact position. It may be contact burn-out or fusing closed, depending upon the failure mode affecting the operation.)

$\mathcal{K}_1$ = an event in which the switch makes contact closure upon the failure of the control function

$\mathcal{K}_2$ = an event in which the switch is opened upon the failure of the control function and which is equal to $\overline{\mathcal{K}_1}$

The events $\mathcal{K}_1$ and $\mathcal{K}_2$ are mutually exclusive and complementary. According to the above definitions, a reliability diagram for this configuration can be immediately constructed and is given in Fig. 8. It is to be noted that $\mathcal{K}_i$ is a subset of $\overline{\mathcal{C}}$ and that the product $\mathcal{K}_i \cdot \overline{\mathcal{C}} = \mathcal{K}_i$. However, the block $\overline{\mathcal{C}}$ will be shown in the reliability diagram for clarity. The use of the block $\overline{\mathcal{C}}$ emphasizes the partial success of this configuration. Similar notation will be used in other configurations, and this statement will not be repeated.

The success event of this configuration is given by

$$\mathcal{S} = \mathcal{K} \cdot [\mathcal{C} + \overline{\mathcal{C}} \cdot \mathcal{K}_i] \tag{19}$$

and the probability of success is given by

$$\Pr[\mathcal{S}] = \Pr[\mathcal{K}] \cdot \{\Pr[\mathcal{K}_i] + \Pr[\mathcal{C}] \cdot \Pr[\overline{\mathcal{K}_i}]\} \tag{20}$$

A plot of Pr $[\mathcal{S}]$ versus Pr $[\mathcal{C}]$ is given in Fig. 9. It can be seen that even though the control function fails completely, the system still has a finite probability of success. It should be pointed out that the switching

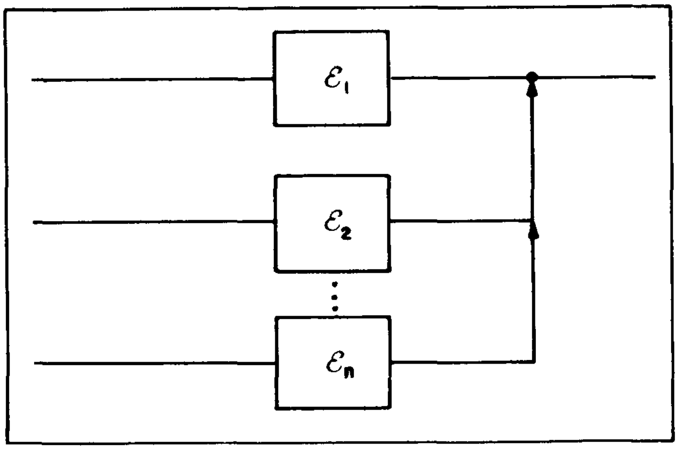


Fig. 4. Stand-by configuration

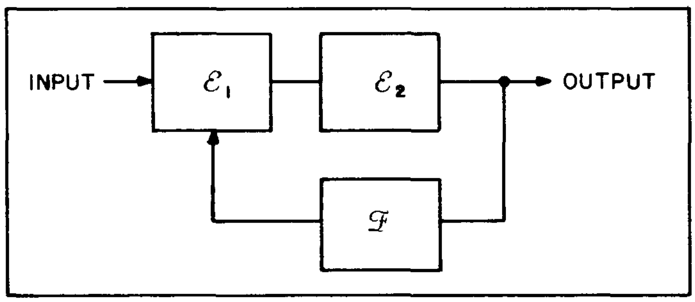


Fig. 5. Feedback control configuration

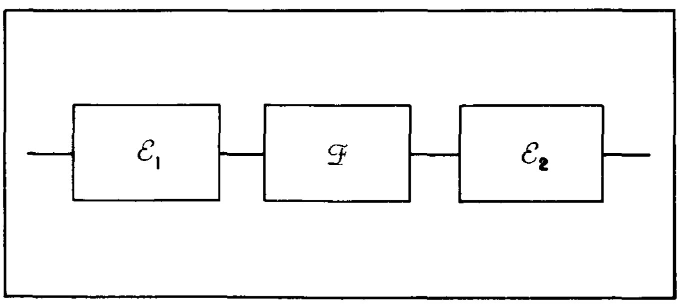


Fig. 6. Equivalent reliability diagram for feedback control configuration

model described previously is suitable for certain switches, while it may not be applicable for others. Different models must be established according to the characteristics of the switches. However, this configuration is reasonably representative of most switch types used in the *Mariner B* spacecraft communication system.

6. Double-Contact Controlled Switching Configuration

The double-contact switch configuration (Fig. 10) bears a remarkable resemblance to the single-contact switch configuration previously described. The only parameters required to be redefined are the two conditional events $\mathcal{K}_1$ and $\mathcal{K}_2$:

$\mathcal{K}_1$ = the event that the switch contact is in the position for which the output is connected to input 1 upon failure of the control function

$\mathcal{K}_2$ = the event that the switch contact is in the position for which the output is connected to input 2 upon failure of the control function

The events $\mathcal{K}_1$ and $\mathcal{K}_2$ are mutually exclusive and complementary.

Figure 11 shows the reliability diagram of this switching configuration. It can be seen from Fig. 11 that if the control function is operating properly, the reliability of this configuration depends entirely on the switch. However, if the control function fails, the switch contact will rest on either one of the two positions representing partial success of the system. This configuration is considered a parallel combination only when the control function is operating properly. The reliability for this configuration is

$$\mathcal{S} = \mathcal{K} \cdot [(\mathcal{C} + \bar{\mathcal{C}} \cdot \mathcal{K}_1) \cdot \mathcal{B}_1 + (\mathcal{C} + \bar{\mathcal{C}} \cdot \mathcal{K}_2) \cdot \mathcal{B}_2] \tag{21}$$

where

$\mathcal{B}_1$ = success event for input 1

$\mathcal{B}_2$ = success event for input 2

The probability of success is

$$\begin{aligned} \Pr[\mathcal{S}] = \Pr[\mathcal{K}] \cdot \Big[& \Pr[\mathcal{C}] \cdot \{\Pr[\mathcal{B}_1] \cdot \Pr[\mathcal{K}_2] + \Pr[\mathcal{B}_2] \cdot \Pr[\mathcal{K}_1] - \Pr[\mathcal{B}_1 \cdot \mathcal{B}_2]\} \\ & + \Pr[\mathcal{B}_1] \cdot \Pr[\mathcal{K}_1] + \Pr[\mathcal{B}_2] \cdot \Pr[\mathcal{K}_2] \Big] \end{aligned} \tag{22}$$

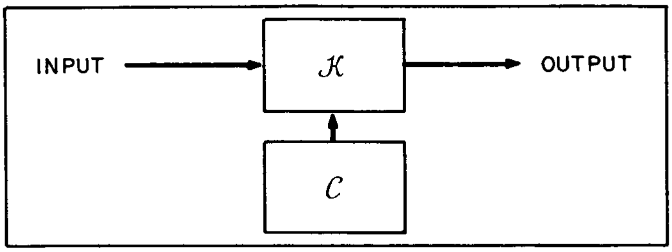


Fig. 7. Simple controlled switching configuration

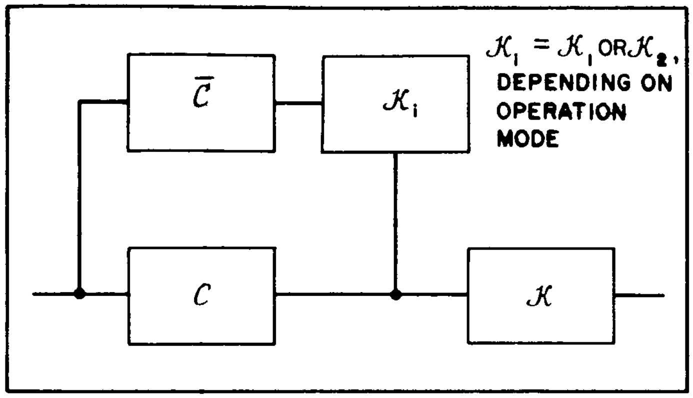


Fig. 8. Equivalent reliability diagram for a simple switching configuration

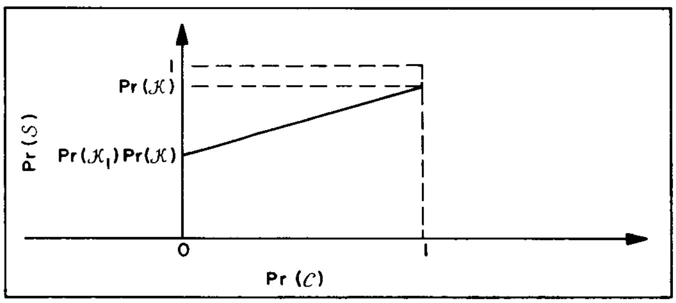


Fig. 9. Pr [$\mathcal{S}$] versus Pr [$\mathcal{C}$]

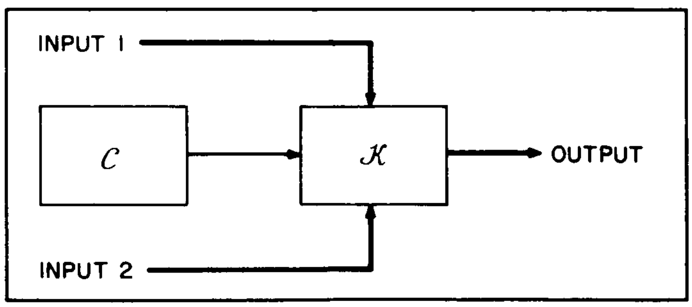


Fig. 10. Double-contact controlled switching configuration

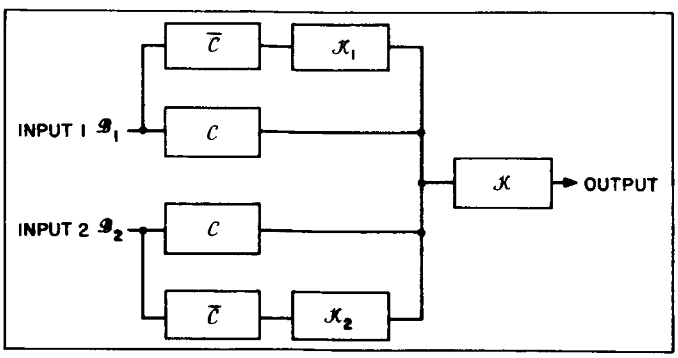


Fig. 11. Reliability diagram for double-contact switching configuration

Since this switching mechanism is bilateral, these equations are also valid for the single input and branching outputs configuration.

7. Cross-Connection Switching Pair Configuration

A double-pole double-throw switch is interconnected as shown in Fig. 12. The switches $\mathcal{K}_1$ and $\mathcal{K}_2$ are ganged together and are energized by a bistable memory element $\mathfrak{M}$, which is controlled by the control function $\mathcal{C}$. To analyze this mechanism, only the following events are considered, with all the other possibilities neglected:

$\mathcal{B}_1$, $\mathcal{B}_2$ = success events for the input functions associated with this switching network

$\mathcal{B}_3$, $\mathcal{B}_4$ = success events of the equipment connected at the outputs of this switch configuration

$\mathcal{C}$ = success event of the control function

$\mathcal{K}_1$ = success event of switch $\mathcal{K}_1$

$\mathcal{K}_2$ = success event of switch $\mathcal{K}_2$

$\mathfrak{M}$ = success event of the bistable memory element

$\mathfrak{M}_{11}$ = event in which the output of the element $\mathfrak{M}$ causes the switches to make contact with one position upon the failure of $\mathfrak{M}$

$\mathfrak{M}_{12}$ = event in which the output of the element $\mathfrak{M}$ causes the switches to make contact with the other position upon the failure of $\mathfrak{M}$

$\mathfrak{M}_{21}$ = event in which the switches make contact with one position upon the failure of the control function $\mathcal{C}$

$\mathfrak{M}_{22}$ = event in which the switches make contact with the other position upon the failure of the control function $\mathcal{C}$

By definition, the control function is said to be perfect if both $\mathfrak{M}$ and $\mathcal{C}$ are operating perfectly. The events $\mathfrak{M}$, $\mathfrak{M}_{11}$, and $\mathfrak{M}_{12}$ are mutually exclusive of one another, and the events $\mathfrak{M}_{21}$ and $\mathfrak{M}_{22}$ are mutually exclusive and complementary. The terms $\mathcal{B}_1$, $\mathcal{B}_2$, $\mathcal{B}_3$, and $\mathcal{B}_4$ must be included in the reliability equation because the switching configuration is not entirely independent of the input and output circuits.

The reliability diagram of the switching complex may be represented as in Fig. 13. The reliability equation for this configuration may be written in one of two forms:

$$\mathcal{S} = \{ \mathcal{B}_1 \cdot [\mathcal{C} \cdot (\mathcal{M} + \mathcal{M}_{11}) + \bar{\mathcal{C}} \cdot \mathcal{M}_{21}] + \mathcal{B}_2 \cdot K_2 \cdot [\mathcal{C} \cdot (\mathcal{M} + \mathcal{M}_{12}) + \bar{\mathcal{C}} \cdot \mathcal{M}_{22}] \} \cdot K_1 \cdot \mathcal{B}_3$$

$$+ \{ \mathcal{B}_2 \cdot [\mathcal{C} \cdot (\mathcal{M} + \mathcal{M}_{11}) + \bar{\mathcal{C}} \cdot \mathcal{M}_{21}] + \mathcal{B}_1 \cdot K_1 \cdot [\mathcal{C} \cdot (\mathcal{M} + \mathcal{M}_{12}) + \bar{\mathcal{C}} \cdot \mathcal{M}_{22}] \} K_2 \cdot \mathcal{B}_4 \qquad (23)$$

or

$$\mathcal{S} = \{ \mathcal{B}_3 \cdot [\mathcal{C} \cdot (\mathcal{M} + \mathcal{M}_{11}) + \bar{\mathcal{C}} \cdot \mathcal{M}_{21}] + \mathcal{B}_4 \cdot K_2 \cdot [\mathcal{C} \cdot (\mathcal{M} + \mathcal{M}_{12}) + \bar{\mathcal{C}} \cdot \mathcal{M}_{22}] \} \cdot K_1 \cdot \mathcal{B}_1$$

$$+ \{ \mathcal{B}_4 \cdot [\mathcal{C} \cdot (\mathcal{M} + \mathcal{M}_{11}) + \bar{\mathcal{C}} \cdot \mathcal{M}_{21}] + \mathcal{B}_3 \cdot K_1 \cdot [\mathcal{C} \cdot (\mathcal{M} + \mathcal{M}_{12}) + \bar{\mathcal{C}} \cdot \mathcal{M}_{22}] \} \cdot K_2 \cdot \mathcal{B}_2 \qquad (24)$$

The selection of Eq. (23) or (24) depends upon the complexity of the expressions $\mathcal{B}_1$, $\mathcal{B}_2$, $\mathcal{B}_3$, and $\mathcal{B}_4$. If $\mathcal{B}_1$ and $\mathcal{B}_2$ are relatively simple compared to $\mathcal{B}_3$ and $\mathcal{B}_4$, Eq. (23) should be selected; otherwise, the proper choice would be Eq. (24). If the complexity of these expressions is of the same order, either equation may be used. The probability of success for this function Pr $[\mathcal{S}]$ may be obtained by applying various probability theorems and will not be given here.

The seven basic configurations described previously cover most of the configurations of the telecommunication system in the *Mariner B* spacecraft as well as the configurations of many other complex systems. By using these basic groups, the analysis of complex system reliability becomes less formidable.

C. *Mariner B* Telecommunication System Block Diagram

The *Mariner B* spacecraft communication system consists of three major components: the radio subsystem, the command subsystem, and the telemetry subsystem. The over-all block diagram describing the interconnection of these three groups is illustrated in Fig. 14.

D. Command Function Reliability Analysis

The simplest form of the command function reliability diagram is shown in Fig. 15. This diagram is similar to the feedback control configuration given in Fig. 5. The signal acquisition function consists of antennas, RF switches, and receivers which acquire, detect, and demodulate the command signals for the command decoder. The external dependents are those functions within the spacecraft which have direct effects upon the communication system. The main power and the attitude-control functions will be included under this block. The command detection and decoding function consists of all the major circuits in the command subsystem for the detection and decoding of the composite signals derived by the receivers. The feedback control function includes the circuits used for radio commands. The external dependents may be

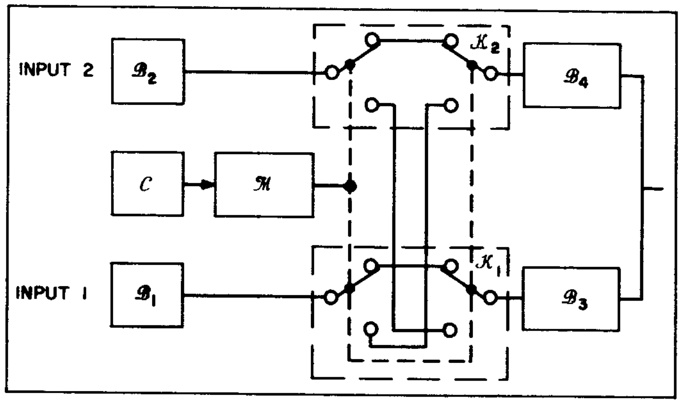


Fig. 12. Schematic diagram for cross connection of switching pair

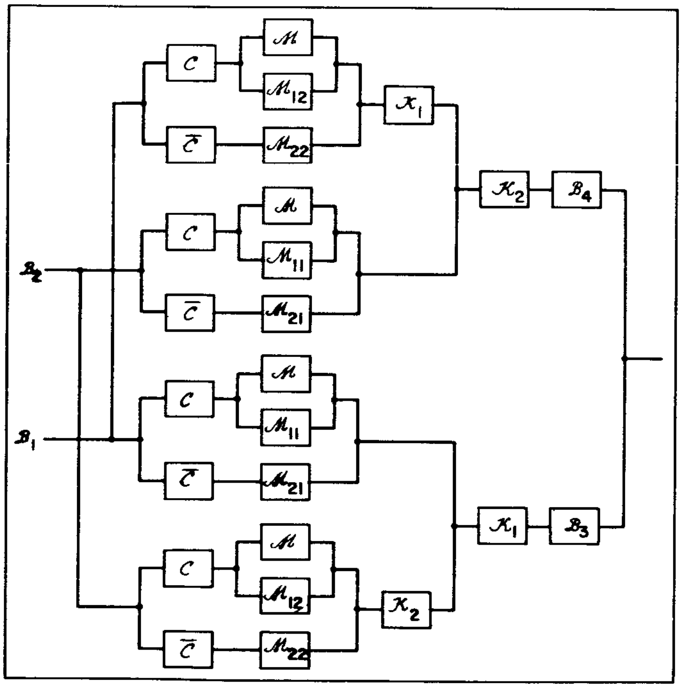


Fig. 13. Reliability diagram for cross connection of switching pair

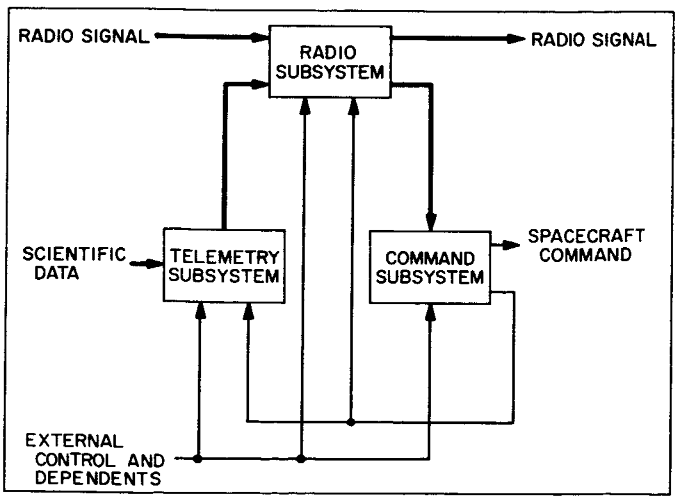


Fig. 14. Over-all block diagram for the *Mariner B* telecommunication system

considered as the functions from other portions of the spacecraft such as the Canopus acquisition and the antenna positioning functions. In addition, external controls are provided for the radio subsystem from the central computer and sequencer (CC&S).

The following symbols are used to denote the success events of the various functions:

C = success event of the command function

C_D = success event of the command detection and decoding function

E = success event of the external dependents

S_W = success event of the signal acquisition for the command function (S_W is also a function of an external control **CCS** and an internal feedback function SK.)

The success event of the command function C may be given by

$$C = \mathbf{E} \cdot C_D \cdot S_W \tag{25}$$

Since these three functions **E**, C_D, and S_W are independent, their probabilities of success may be obtained separately.

1. Success Event of the External Dependents

The success event **E** is

$$\mathbf{E} = (\mathbf{PO}) \cdot (\mathbf{AC}) \tag{26}$$

and the associated probability is

$$\Pr[\mathbf{E}] = \Pr[(\mathbf{PO}) \cdot (\mathbf{AC})] \tag{27}$$

where

(**PO**) = success event of main power

(**AC**) = success event of attitude control

Although the two functions **PO** and **AC** are not independent, as a simplification they are assumed to be independent in the communication system analysis.

2. Success Event C_D

The success event C_D is obtained by first constructing the functional and reliability diagrams which are shown in Fig. 16 and 17, respectively. The following symbols are used to denote the success events of the various components:

G_1 = success event of the command input amplifier

G_2 = success event of the command detector 1

G_3 = success event of the command detector 2

G_4 = success event of the power selector

G_5 = success event of the decoder access switch

G_6 = success event of the command decoder

P_1 = success event of the transformer-rectifier unit for command detector 1

P_2 = success event of the transformer-rectifier unit for command detector 2

P_3 = success event of either P_1 or P_2 and the power selector G_4

S_1 = success event of the selector S_1

S_{11} = event of the switch making contact with command detector 1 when the selector has failed

S_{12} = event of the switch making contact with command detector 2 when the selector has failed

The events S_1, S_{11}, and S_{12} are mutually exclusive.

Although P_3 supplies the power for most of the circuits in the command subsystem, it is not included in the reliability diagram because P_3 is derived from either P_1 or P_2, determined by the power selector G_4. The relationship of these power supplies is

$$\begin{aligned} & P_3 \cdot [G_2 \cdot P_1 \cdot (S_1 + S_{11}) + G_3 \cdot P_2 \cdot (S_1 + S_{12})] \\ &= G_4 \cdot (P_1 + P_2) \cdot [G_2 \cdot P_1 \cdot (S_1 + S_{11}) + G_3 \cdot P_2 \cdot (S_1 + S_{12})] \\ &= G_4 \cdot [G_2 \cdot P_1 \cdot (S_1 + S_{11}) + G_3 \cdot P_2 \cdot (S_1 + S_{12})] \qquad (28) \end{aligned}$$

It is also obvious that P_3 is only a fictitious event since there is no physical counterpart associated with it.

From Fig. 17, the event equation for the command detection and decoding function becomes apparent:

$$C_D = G_1 \cdot [G_2 \cdot P_1 \cdot (S_1 + S_{11}) + G_3 \cdot P_2 \cdot (S_1 + S_{12})] \cdot G_4 \cdot G_5 \cdot G_6 \tag{29}$$

The probability of success for C_D is

$$\Pr[C_D] = \Pr[G_1 \cdot G_4 \cdot G_5 \cdot G_6] \cdot \Big[\Pr[S_1] \cdot \{\Pr[G_2 \cdot P_1] + \Pr[G_3 \cdot P_2] - \Pr[G_2 \cdot G_3 \cdot P_1 \cdot P_2]\} + \Pr[G_2 \cdot P_1 \cdot S_{11}] + \Pr[G_3 \cdot P_2 \cdot S_{12}]\}\Big] \tag{30}$$

3. Success Event S_W

The success event S_W is obtained by constructing the functional diagram shown in Fig. 18. The definitions for the symbols used are listed as follows:

CCS = success event of central computer and sequencer

AQ = success event of Canopus acquisition

Ap = success event of antenna positioning

A_{HG} = success event of high-gain antenna

A_1 = success event of low-gain antenna 1

A_2 = success event of low-gain antenna (preferred)

P_4 = success event of transformer-rectifier unit for radio subsystem

SK = success event of control switching circuitry

R_1 = success event of receiver 1

R_2 = success event of receiver 2

S_2 = success event of the selector S_2

S_{21} = event of the switch making contact with receiver 1 when selector S_2 has failed

S_{22} = event of the switch making contact with receiver 2 when selector S_2 has failed

K_i = success event of RF switch K_i

K_{i1} = event of RF switch K_i making contact with one direction when the control to this switch has failed

K_{i2} = event of RF switch K_i making contact with the other direction when the control to this switch has failed

M = success event of the memory element controlling the switching pair K_1 and K_2

M_{11} = event in which the switches K_1 and K_2 are caused to make contact with one position when M has failed

M_{12} = event in which the switches K_1 and K_2 are caused to make contact with the other position when M has failed

M_{21} = event in which the switches K_1 and K_2 are locked to one position when the control function to the memory element M has failed

M_{22} = event in which the switches K_1 and K_2 are locked to the other position when the control function to the memory element M has failed

Notice that K_{i1} and K_{i2} are mutually exclusive and complementary. The events M, M_{11}, and M_{12} are mutually exclusive, and M_{21} and M_{22} are mutually exclusive. All other conditions not mentioned here are neglected.

It has been shown that,for a feedback control configuration, it is necessary to include only the feedback mechanism in the control function. In this case, the feedback control consists of the individual control circuits or switches denoted by SK_i. For simplicity, it is assumed that all the control circuits are operating as a single integral part, and the symbol SK is selected as the success event of the feedback control function. External control functions include central computer and sequencer (**CCS**), Canopus acquisition (**AQ**), and antenna positioning (**Ap**), which control the operation of the high-gain antenna. In addition, **CCS** serves as a backup command to various RF switches. The RF switches K_1 and K_2 are connected as a cross-connection switching pair similar to the one described in Fig. 12. A detailed reliability diagram for this function can now be prepared and is shown in Fig. 19.

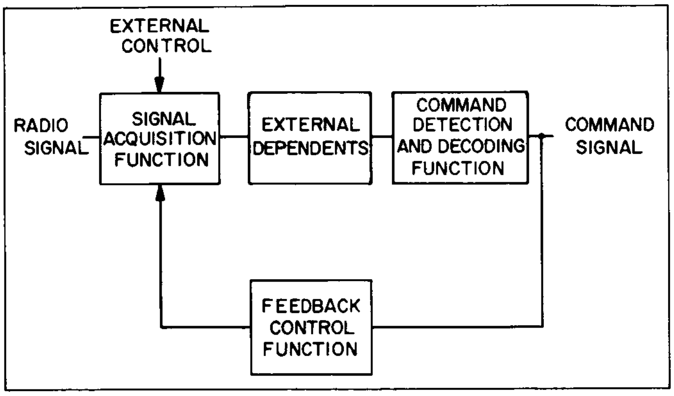


Fig. 15. Reliability diagram for command function

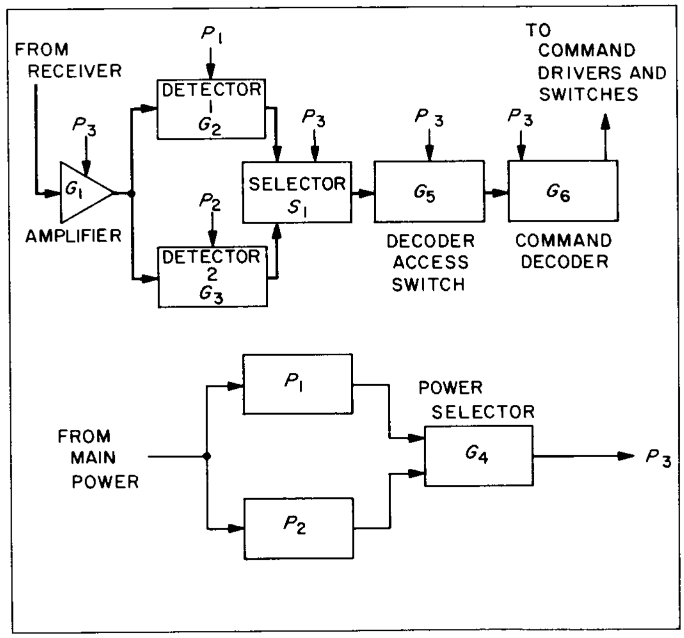


Fig. 16. Command encoding and decoding functional block diagram

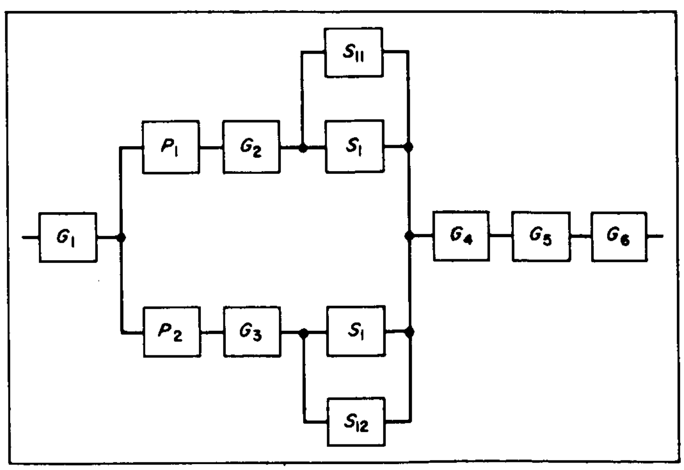


Fig. 17. Reliability diagram for command encoding and decoding function

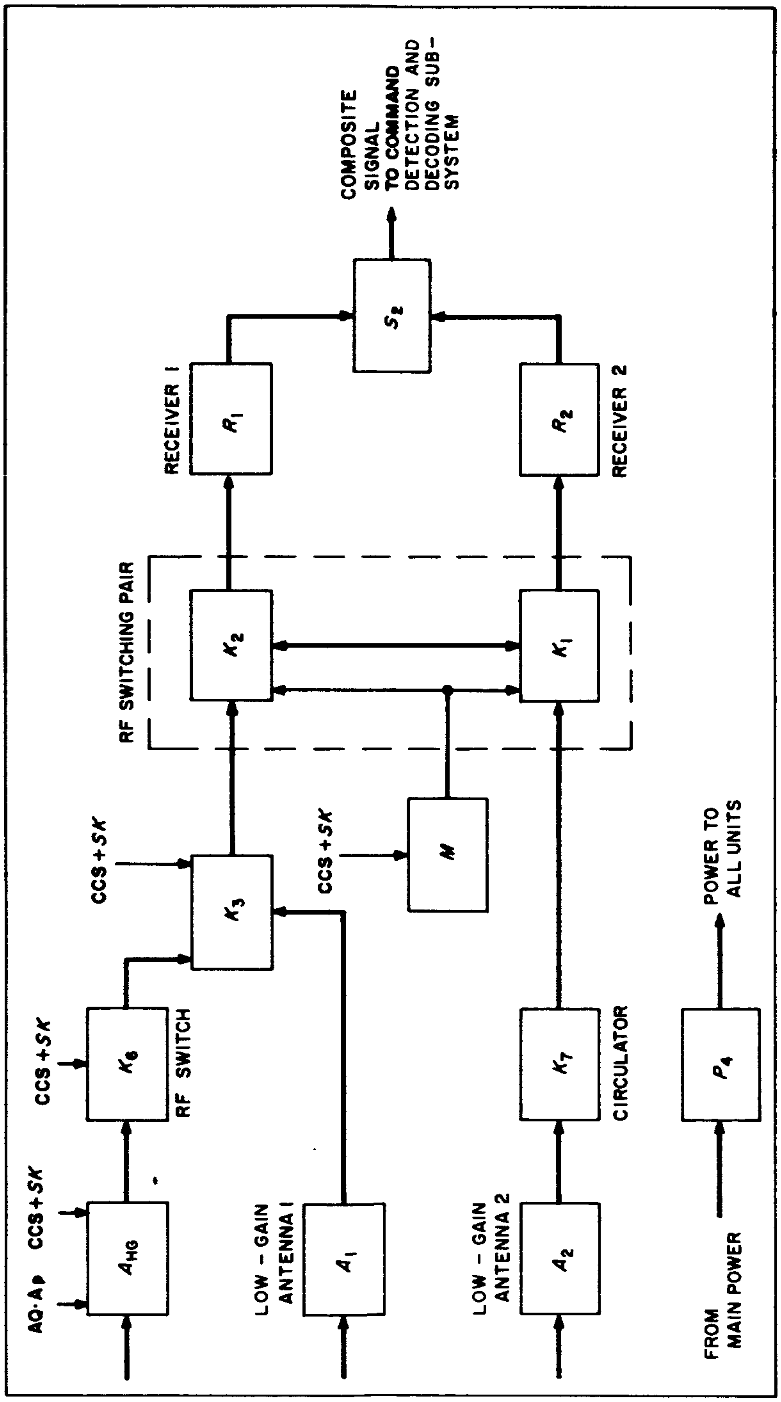


Fig. 18. Signal acquisition functional diagram

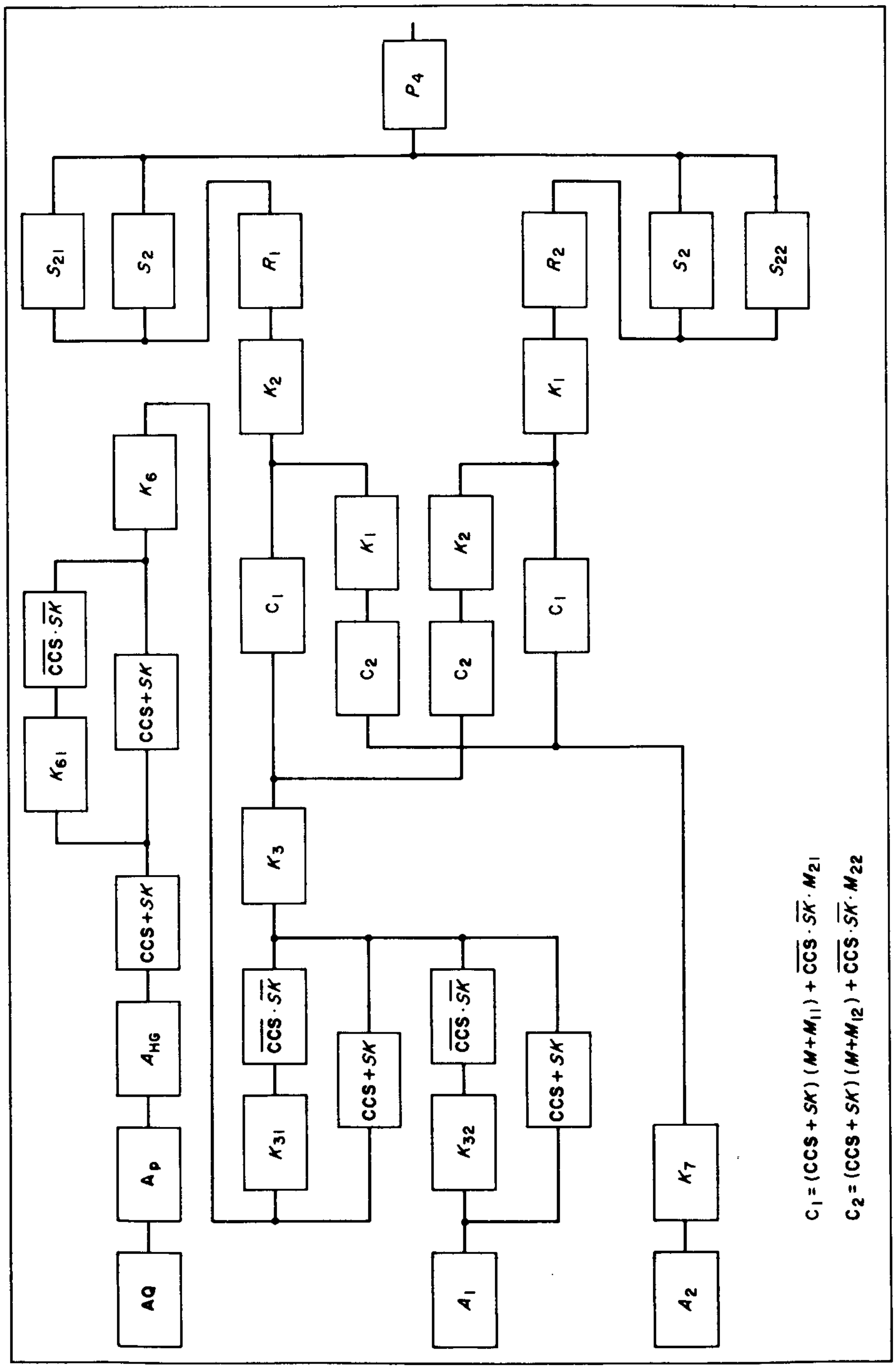


Fig. 19. Signal acquisition function reliability diagram

To derive the success event equation for S_W, it is convenient to write the equation around the cross-connection switching pair. By following Eq. (24), the event equation for the signal acquisition function is

$$\begin{aligned} S_W = P_4 \cdot \Big[& \{B_3 \cdot [(\mathbf{CCS} + SK) \cdot (M + M_{11}) + \overline{\mathbf{CCS}} \cdot \overline{SK} \cdot M_{21}] \\ & + B_4 \cdot [(\mathbf{CCS} + SK) \cdot (M + M_{12}) + \overline{\mathbf{CCS}} \cdot \overline{SK} \cdot M_{22}] \cdot K_2\} \cdot K_1 \cdot B_1 \\ & + \{B_4 \cdot [(\mathbf{CCS} + SK) \cdot (M + M_{11}) + \overline{\mathbf{CCS}} \cdot \overline{SK} \cdot M_{21}] \\ & + B_3 \cdot [(\mathbf{CCS} + SK) \cdot (M + M_{12}) + \overline{\mathbf{CCS}} \cdot \overline{SK} \cdot M_{22}] \cdot K_1\} \cdot K_2 \cdot B_2 \Big] \end{aligned} \tag{31}$$

where

$$B_1 = A_2 \cdot K_7 \tag{32}$$

$$B_2 = (\mathbf{AQ} \cdot \mathbf{Ap}) \cdot A_{HG} \cdot (\mathbf{CCS} + SK) \cdot K_3 \cdot K_6 + A_1 \cdot [(\mathbf{CCS} + SK) + \overline{\mathbf{CCS}} \cdot \overline{SK} \cdot K_{32}] \cdot K_3 \tag{33}$$

$$B_3 = R_2 \cdot (S_2 + S_{22}) \tag{34}$$

$$B_4 = R_1 \cdot (S_2 + S_{21}) \tag{35}$$

Substituting the B_i's into Eq. (31) yields

$$\begin{aligned} S_W = P_4 \cdot \Big[& \{R_2 \cdot (S_2 + S_{22}) \cdot [(\mathbf{CCS} + SK) \cdot (M + M_{11}) + \overline{\mathbf{CCS}} \cdot \overline{SK} \cdot M_{21}] \\ & + R_1 \cdot (S_2 + S_{21}) \cdot [(\mathbf{CCS} + SK) \cdot (M + M_{12}) + \overline{\mathbf{CCS}} \cdot \overline{SK} \cdot M_{22}] \cdot K_2\} \cdot K_1 \cdot A_2 \cdot K_7 \\ & + \{R_1 \cdot (S_2 + S_{21}) \cdot [(\mathbf{CCS} + SK) \cdot (M + M_{11}) + \overline{\mathbf{CCS}} \cdot \overline{SK} \cdot M_{21}] \\ & + R_2 \cdot (S_2 + S_{22}) \cdot [(\mathbf{CCS} + SK) \cdot (M + M_{12}) + \overline{\mathbf{CCS}} \cdot \overline{SK} \cdot M_{22}] \cdot K_1\} \\ & \cdot K_2 \cdot \{(\mathbf{AQ}) \cdot \mathbf{Ap} \cdot A_{HG} \cdot (\mathbf{CCS} + SK) \cdot K_3 \cdot K_6 \\ & + A_1 \cdot [(\mathbf{CCS} + SK) + \overline{\mathbf{CCS}} \cdot \overline{SK} \cdot K_{32}] \cdot K_3\} \Big] \end{aligned} \tag{36}$$

The derivation of the equation Pr $[S_W]$ is given in Appendix A. The probability of success of the function S_W has the following form:

$$\mathrm{Pr}\ [S_W] = y_1 + y_2\ \mathrm{Pr}\ [\mathbf{CCS}] + y_3\ \mathrm{Pr}\ [\mathbf{AQ} \cdot \mathbf{Ap}] + y_4\ \mathrm{Pr}\ [\mathbf{CCS}] \cdot \mathrm{Pr}\ [\mathbf{AQ} \cdot \mathbf{Ap}] \tag{37}$$

where the y_i's are constants in terms of the probabilities of success of various events within the sample space of the signal acquisition function. A similar form is expected for the equation of the command function reliability:

$$\mathrm{Pr}\ [C] = \mathrm{Pr}\ [\mathbf{E}] \cdot \mathrm{Pr}\ [C_D] \cdot \mathrm{Pr}\ [S_W] = y_1' + y_2'\ \mathrm{Pr}\ [\mathbf{CCS}] + y_3'\ [\mathbf{AQ} \cdot \mathbf{Ap}]$$

$$+ y_4'\ \mathrm{Pr}\ [\mathbf{CCS}] \cdot \mathrm{Pr}\ [\mathbf{AQ} \cdot \mathbf{Ap}] \tag{38}$$

A three-dimensional plot for Eq. (38) may be performed with Pr $[C]$ as the dependent variable and Pr [**CCS**] and Pr [**AQ** · **Ap**] as the independent variables. Figure 20 demonstrates the general structure of this three-dimensional plot. This diagram provides a quick estimate of the probability of success of the command function for each given pair of Pr [**CCS**] and Pr [**AQ** · **Ap**]. Although the surface Pr $[C]$ is not necessarily a plane, for every constant Pr [**CCS**] or Pr [**AQ** · **Ap**], straight lines on the surface are expected since the surface is defined by Eq. (38). A measure of the vertical line drawn from the selected pair Pr [**CCS**] and Pr [**AQ** · **Ap**] on the horizontal plane to the corresponding point on the surface Pr $[C]$ gives the probability of success of the command function related to this given pair of Pr [**CCS**] and Pr [**AQ** · **Ap**].

The construction of this diagram is simple. One needs to obtain only the four corners α, β, γ, and δ on the surface in order to describe fully the characteristics of the surface. It can be seen from Eq. (38) that the corners are

$$\alpha = y_1' + y_2' + y_3' + y_4'$$

$$\beta = y_1' + y_3'$$

$$\gamma = y_1' + y_2'$$

$$\delta = y_1'$$

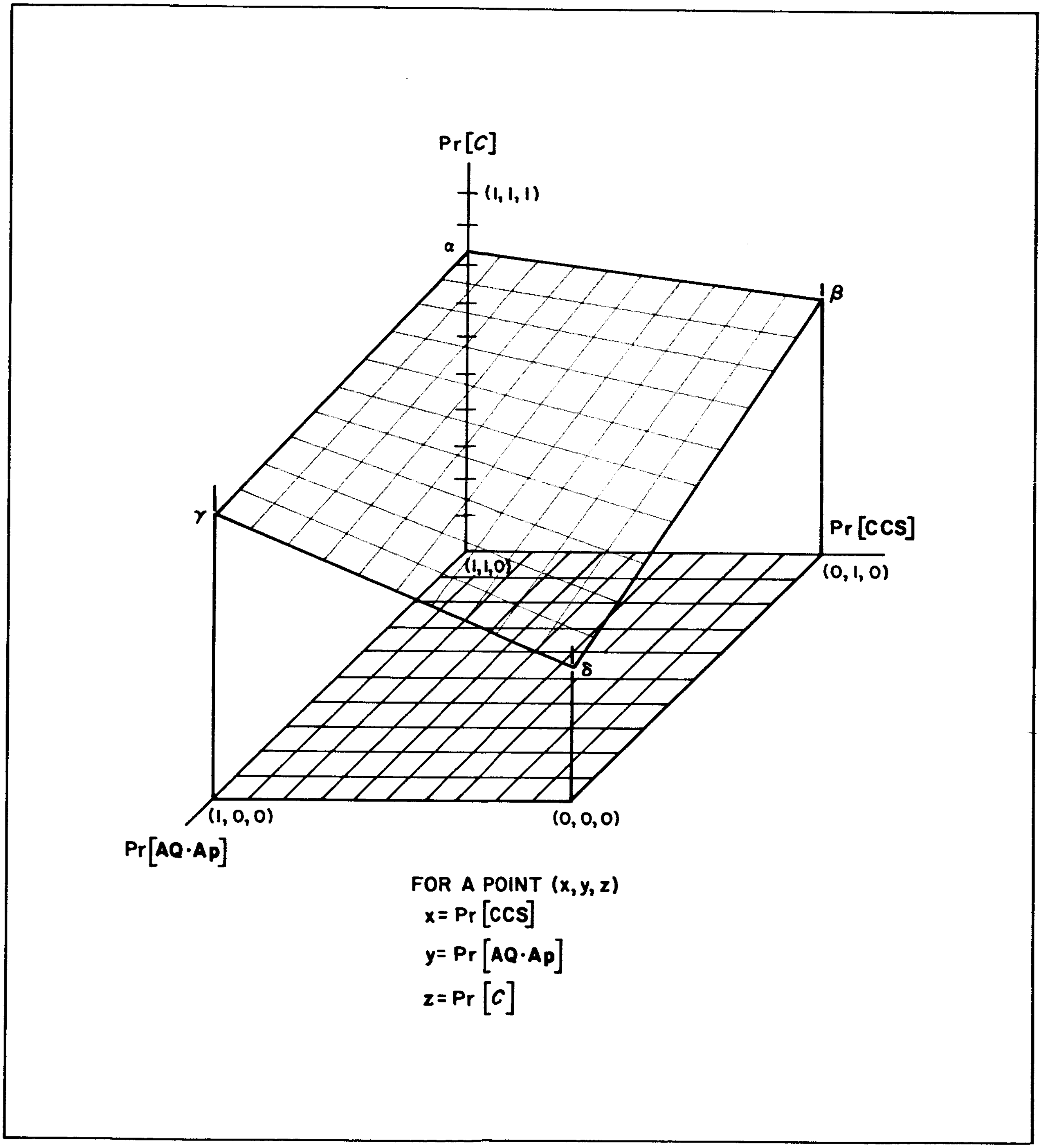


Fig. 20. Three-dimensional plot of Pr [C] versus Pr [**CCS**] and Pr [**AQ** · **Ap**]

E. Two-Way Doppler Function Reliability Analysis

The two-way doppler function reliability diagram may be represented by Fig. 21. Although there are five main blocks in the diagram, the signal acquisition and the signal transmission functions may be treated as a single unit since the antennas and RF switches are common to both functions. The transmission preparation function includes the necessary transmitters and power amplifiers to bring the signal to a suitable level for transmission. The external dependents and the command detection and decoding function are identical to those described in the command function analysis. However, the latter is not treated as a series block since the failure of this function would not directly cause the total failure of the two-way doppler function.

The success event of the two-way doppler function is

$$D_T = \mathbf{E} \cdot (S_X \cdot T) \tag{39}$$

where

$$(S_X \cdot T) = f\,(C_D', \text{external control}) \tag{40}$$

Here

D_T = success event of the two-way doppler function

$\mathbf{E}$ = success event of the external dependents

S_X = success event of the signal acquisition and transmission functions

T = success event of the transmission preparation function

C_D' = success event of the command detection and decoding function, including the reliability of various control switches

The term $S_X \cdot T$ is in parentheses, signifying that the two functions S_X and T are not independent: they must be grouped together when the probability of success for this term is evaluated. However, the event equations for the two functions may be written separately. The success events of $\mathbf{E}$ and C_D' may be immediately established:

$$\mathbf{E} = (\mathbf{PO}) \cdot (\mathbf{AC})$$

$$C_D' = C_D \cdot SK'$$

1. Success Event T

The success event T can be obtained by constructing the functional diagram shown in Fig. 22. Two transmitters, G_7 and G_8, are provided and are powered by the radio subsystem supply P_4 through a selecting switch S_3. Ordinarily only one transmitter will be energized at any given instant. The selected power also operates the RF switch to connect the proper transmitter output to the power amplifier. The transmission preparation function also includes two power amplifiers (amplitrons), G_9 and G_{10}, connected in cascade. These amplitrons are powered by a separate power supply P_7 through an independent selecting switch S_4 such that only one amplitron is energized at any time. When energized, the amplitron provides power amplification of the input signal; otherwise, it serves as a transmission line with insignificant insertion loss. As far as the reliability diagram is concerned, the two amplitrons are operated in parallel as amplifiers and are operated in series as low-loss transmission lines. The reliability diagram for this function can now be constructed and is shown in Fig. 23.

The symbols used in Fig. 23 are defined as follows:

G_7 = success event of transmitter 1, including transfer switch

G_8 = success event of transmitter 2, including transfer switch

S_3 = success event of the transmitter power-selecting switch

K_4 = success event of the RF switch K_4

$(S_3 \cdot K_4)_1$ = event that both S_3 and K_4 are in the proper positions for transmitter 1 to make connection with the amplitron input when control has failed

$(S_3 \cdot K_4)_2$ = event that both S_3 and K_4 are in the proper positions for transmitter 2 to make connection with the amplitron input when control has failed

G_9 = success event of amplitron 1 as an amplifier when energized

G_9' = success event of amplitron 1 as a transmission line when de-energized

G_{10} = success event of amplitron 2 as an amplifier when energized

G'_{10} = success event of amplitron 2 as a transmission line when de-energized

S_4 = success event of amplitron power-selecting switch

S_{41} = event that S_4 makes contact with amplitron 1 when control has failed

S_{42} = event that S_4 makes contact with amplitron 2 when control has failed

The events $(S_3 \cdot K_4)_1$ and $(S_3 \cdot K_4)_2$ are mutually exclusive; S_{41} and S_{42} are mutually exclusive. All other conditions for the switches are omitted.

The success event for T is

$$T = \{[C'_D + \overline{C'_D} \cdot (S_3 \cdot K_4)_1] \cdot G_7 + [C'_D + \overline{C'_D} \cdot (S_3 \cdot K_4)_2] \cdot G_8\} \cdot S_3 \cdot K_4$$

$$\cdot \{[C'_D + \overline{C'_D} \cdot S_{41}] \cdot G_9 + [C'_D + \overline{C'_D} \cdot S_{42}] \cdot G_{10}\} \cdot S_4 \cdot G'_9 \cdot G'_{10} \cdot P_7$$

$$= G'_9 \cdot G'_{10} \cdot K_4 \cdot P_7 \cdot S_3 \cdot S_4 \cdot \{C'_D \cdot (G_7 + G_8)(G_9 + G_{10}) + \overline{C'_D} \cdot [(S_3 \cdot K_4)_1 \cdot G_7$$

$$+ (S_3 \cdot K_4)_2\, G_8] \cdot [S_{41} \cdot G_9 + S_{42} \cdot G_{10}]\} \qquad (41)$$

2. Success Event S_X

Figure 24 shows the functional diagram of the signal transmission function. The combination of Fig. 18 and Fig. 24 forms the sample space of the function S_X. Note that the command inputs become C'_D instead of SK as denoted in Fig. 18. The reliability diagram for this function is given in Fig. 25.

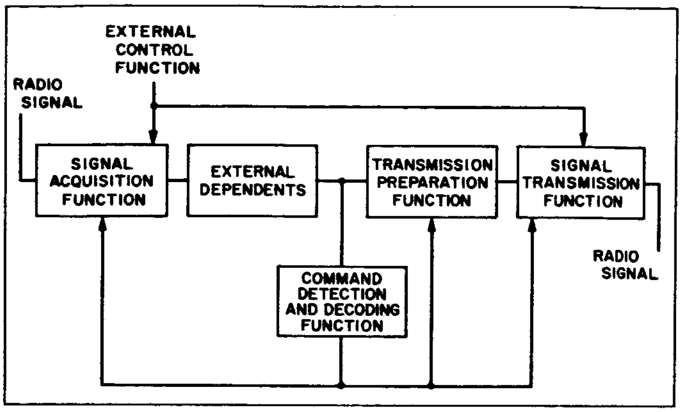


Fig. 21. Two-way doppler function reliability diagram

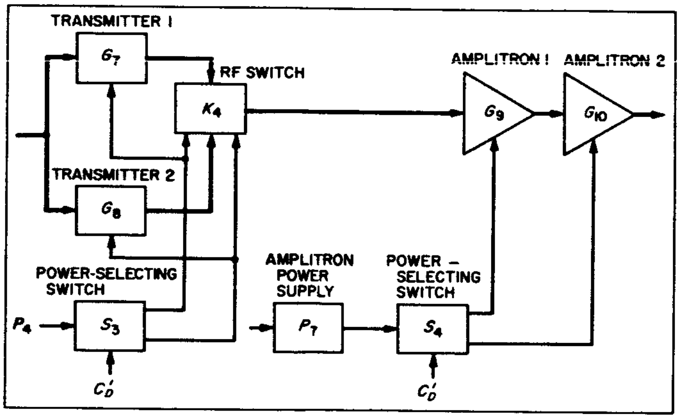


Fig. 22. Functional block diagram for transmission preparation function

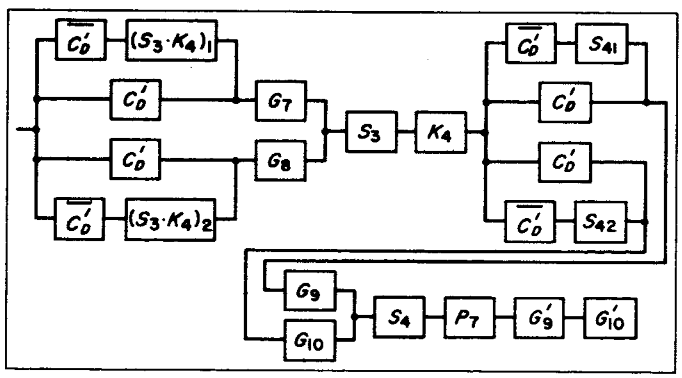


Fig. 23. Reliability diagram for transmission preparation function

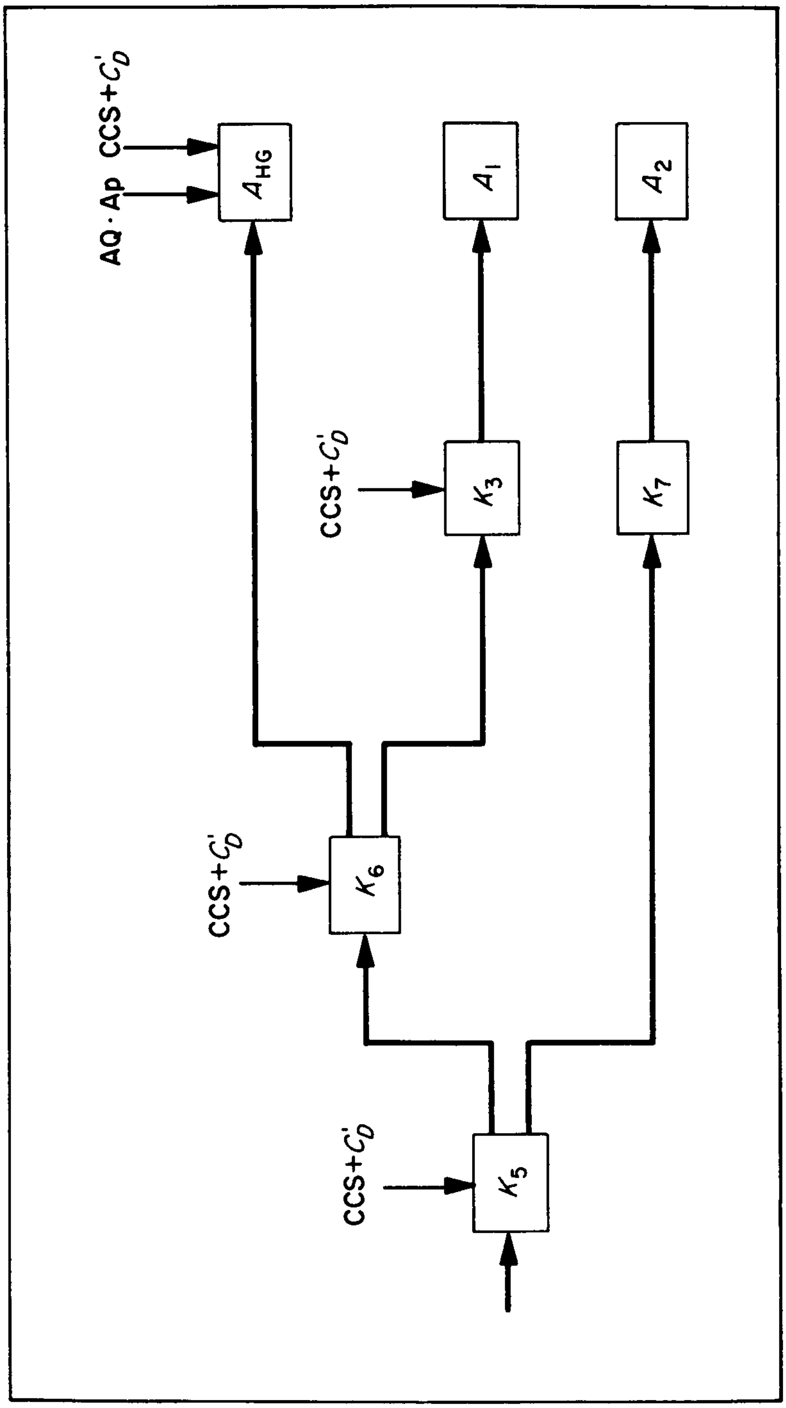


Fig. 24. Signal transmission functional diagram

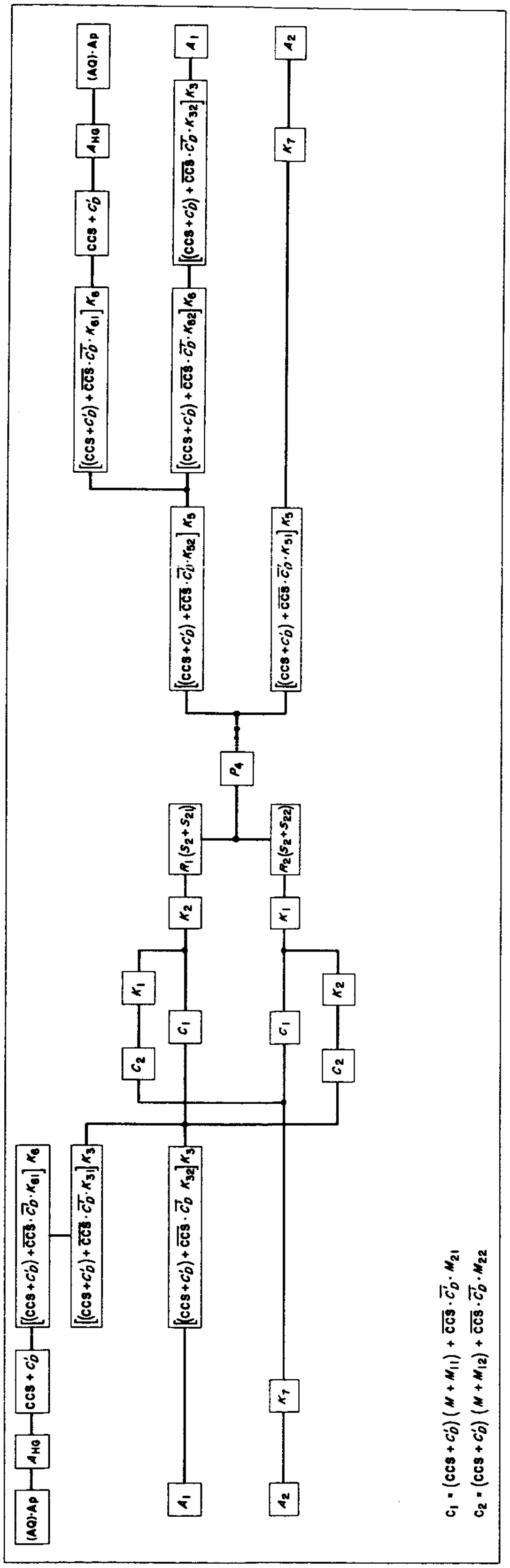


Fig. 25. Reliability diagram for signal acquisition and transmission function

Because of the switching limitation, there are only eight possible two-way transmission paths available. These available paths are tabulated in Table 1.

Table 1. Available two-way transmission paths

Path	Receiving antenna	Transmitting antenna
1	A_1	A_1
2	A_1	A_2
3	A_1	A_{HG}
4	A_2	A_1
5	A_2	A_2
6	A_2	A_{HG}
7	A_{HG}	A_2
8	A_{HG}	A_{HG}

The function S_X is then limited by these eight paths.

For paths 4, 5, and 6,

$$S_{X_1} = \mathrm{B}_1 \cdot [(\mathrm{B}_3 \cdot \mathrm{C}_1 + \mathrm{B}_4 \cdot \mathrm{C}_2 \cdot K_2) \cdot K_1] \cdot (\mathrm{L}_1 \cdot \mathrm{B}_5 + \mathrm{L}_2 \cdot \mathrm{B}_6) \cdot P_4 \tag{42}$$

For path 1,

$$S_{X_2} = \mathrm{B}_{21} \cdot [(\mathrm{B}_4 \cdot \mathrm{C}_1 + \mathrm{B}_3 \cdot \mathrm{C}_2 \cdot K_1) \cdot K_2] \cdot (\mathrm{L}_2 \cdot \mathrm{B}_{62}) \cdot P_4 \tag{43}$$

For paths 2, 3, 7, and 8,

$$S_{X_3} = \mathrm{B}_2 \cdot [(\mathrm{B}_4 \cdot \mathrm{C}_1 + \mathrm{B}_3 \cdot \mathrm{C}_2 \cdot K_1) \cdot K_2] \cdot (\mathrm{L}_1 \cdot \mathrm{B}_5 + \mathrm{L}_2 \cdot \mathrm{B}_{61}) \cdot P_4 \tag{44}$$

The function S_X may be obtained by combining Eq. (42) to (44):

$$S_X = S_{X_1} + S_{X_2} + S_{X_3} \tag{45}$$

The symbols used above may be defined as follows:

$$C_1 = (\mathbf{CCS} + C_D') \cdot (M + M_{11}) + \overline{\mathbf{CCS}} \cdot \overline{C_D'} \cdot M_{21} \tag{46}$$

$$C_2 = (\mathbf{CCS} + C_D') \cdot (M + M_{12}) + \overline{\mathbf{CCS}} \cdot \overline{C_D'} \cdot M_{22} \tag{47}$$

$$B_1 = A_2 \cdot K_7 \tag{48}$$

$$B_2 = (\mathbf{AQ}) \cdot \mathbf{Ap} \cdot A_{HG} \cdot K_3 \cdot K_6 \cdot (\mathbf{CCS} + C_D') + A_1 \cdot K_3 \cdot [(\mathbf{CCS} + C_D') + \overline{\mathbf{CCS}} \cdot \overline{C_D'} \cdot K_{32}] \tag{49}$$

$$B_{21} = A_1 \cdot K_3 \cdot [(\mathbf{CCS} + C_D') + \overline{\mathbf{CCS}} \cdot \overline{C_D'} \cdot K_{32}] \tag{50}$$

$$B_5 = A_2 \cdot K_7 = B_1 \tag{51}$$

$$B_6 = (\mathbf{AQ}) \cdot \mathbf{Ap} \cdot A_{HG} \cdot K_6 \cdot (\mathbf{CCS} + C_D') + A_1 \cdot K_3 \cdot K_6 \cdot [(\mathbf{CCS} + C_D') + \overline{\mathbf{CCS}} \cdot \overline{C_D'} \cdot K_{32} \cdot K_{62}] \tag{52}$$

$$B_{61} = (\mathbf{AQ}) \cdot \mathbf{Ap} \cdot A_{HG} \cdot K_6 \cdot (\mathbf{CCS} + C_D') \tag{53}$$

$$B_{62} = A_1 \cdot K_3 \cdot K_6 \cdot [(\mathbf{CCS} + C_D') + \overline{\mathbf{CCS}} \cdot \overline{C_D'} \cdot K_{32} \cdot K_{62}] \tag{54}$$

$$B_3 = R_2 \cdot (S_2 + S_{22}) \tag{55}$$

$$B_4 = R_1 \cdot (S_2 + S_{21}) \tag{56}$$

$$L_1 = K_5 \cdot [(\mathbf{CCS} + C_D') + \overline{\mathbf{CCS}} \cdot \overline{C_D'} \cdot K_{51}] \tag{57}$$

$$L_2 = K_5 \cdot [(\mathbf{CCS} + C_D') + \overline{\mathbf{CCS}} \cdot \overline{C_D'} \cdot K_{52}] \tag{58}$$

It can also be proved that

$$B_{21} \cdot B_{62} = B_{62}$$

$$B_6 \cdot B_{61} = B_{61}$$

$$B_6 \cdot B_{62} = B_{62}$$

With these relations specified,

$$
\begin{aligned}
S_X &= P_4 \cdot \{ [B_3 \cdot C_1 + B_4 \cdot C_2 \cdot K_2) \cdot K_1] \cdot (L_1 + L_2 \cdot B_2) \cdot B_1 \\
&\quad + [(B_4 \cdot C_1 + B_3 \cdot C_2 \cdot K_1) \cdot K_2] \cdot [L_1 \cdot B_1 \cdot B_2 + L_2 \cdot (B_{62} + B_2 \cdot B_{61})] \} \\
&= P_4 \cdot \Big[\{ R_2 \cdot (S_2 + S_{22}) \cdot [(\mathbf{CCS} + C_D') \cdot (M + M_{11}) + \overline{\mathbf{CCS}} \cdot \overline{C_D'} \cdot M_{21}] \\
&\quad + R_1 \cdot (S_2 + S_{21}) \cdot [(\mathbf{CCS} + C_D') \cdot (M + M_{12}) + \overline{\mathbf{CCS}} \cdot \overline{C_D'} \cdot M_{22}] \cdot K_2 \} \cdot K_1 \cdot (L_1 + L_2 \cdot B_2) \cdot B_1 \\
&\quad + \{ R_1 \cdot (S_2 + S_{21}) \cdot [(\mathbf{CCS} + C_D') \cdot (M + M_{11}) + \overline{\mathbf{CCS}} \cdot \overline{C_D'} \cdot M_{21}] \\
&\quad + R_2 \cdot (S_2 + S_{22}) \cdot [(\mathbf{CCS} + C_D') \cdot (M + M_{12}) + \overline{\mathbf{CCS}} \cdot \overline{C_D'} \cdot M_{22}] \cdot K_1 \} \cdot K_2 \\
&\quad \cdot [L_1 \cdot B_1 \cdot B_2 + L_2 \cdot (B_{62} + B_2 \cdot B_{61})] \Big]
\end{aligned}
\tag{59}
$$

Let

$$Q_1 = (L_1 + L_2 \cdot B_2) \cdot B_1 \tag{60}$$

and

$$Q_2 = [L_1 \cdot B_1 \cdot B_2 + L_2 \cdot (B_{62} + B_2 \cdot B_{61})] \tag{61}$$

Substituting Q_1 and Q_2 into and rearranging Eq. (59) yields

$$
\begin{aligned}
S_X = P_4 \cdot \Big[& [R_2 \cdot K_1 \cdot (S_2 + S_{22}) \cdot Q_1 + R_1 \cdot K_2 \cdot (S_2 + S_{21}) \cdot Q_2] \cdot [(\mathbf{CCS} + C_D') \cdot (M + M_{11}) + \overline{\mathbf{CCS}} \cdot \overline{C_D'} \cdot M_{21}] \\
& + [R_1 \cdot K_1 \cdot K_2 \cdot (S_2 + S_{21}) \cdot Q_1 + R_2 \cdot K_1 \cdot K_2 \cdot (S_2 + S_{22}) \cdot Q_2] \cdot [\mathbf{CCS} + C_D') \cdot (M + M_{12}) + \overline{\mathbf{CCS}} \cdot \overline{C_D'} \cdot M_{22}] \Big]
\end{aligned}
\tag{62}
$$

Taking the product of Eq. (62) and (41), one can obtain the joint probability Pr $[S_X \cdot T]$ in terms of the external control functions. The derivation of this probability of success is given in Appendix B. Substituting the results of Pr $[S_X \cdot T]$ into Eq. (40) yields the probability of success of the two-way doppler function. The form of this equation is identical with that of the command function success equation.

$$\Pr\,[D_T] = z_1 + z_2 \Pr\,[\mathbf{CCS}] + z_3 \Pr\,[\mathbf{AQ} \cdot \mathbf{Ap}] + z_4 \Pr\,[\mathbf{CCS}] \cdot \Pr\,[\mathbf{AQ} \cdot \mathbf{Ap}] \tag{63}$$

A similar three-dimensional plot may be constructed for Eq. (63).

F. One-Way Doppler Function Reliability Analysis

Compared to the two-way doppler function, the one-way doppler function may be considered as a poor accuracy back-up feature for certain missions. It should operate when the receivers are not locked to the signal transmitted from the ground station, or when certain portions of the receivers are not operating properly. It is assumed that when the one-way doppler function is in operation, no command signals are available, and the only source of switching control is from the central computer and sequencer. The functional block diagram for the one-way doppler function, excluding the external dependents, is given in Fig. 26.

Subsequently, the reliability diagram for the one-way doppler function can be prepared and is shown in Fig. 27. Because of the relative simplicity of this function, it can be analyzed as a single unit.

In Fig. 26 and 27,

G_{11} = success event of auxiliary oscillator associated with transmitter 1

G_{12} = success event of auxiliary oscillator associated with transmitter 2

All other symbols have been defined in the previous Section.

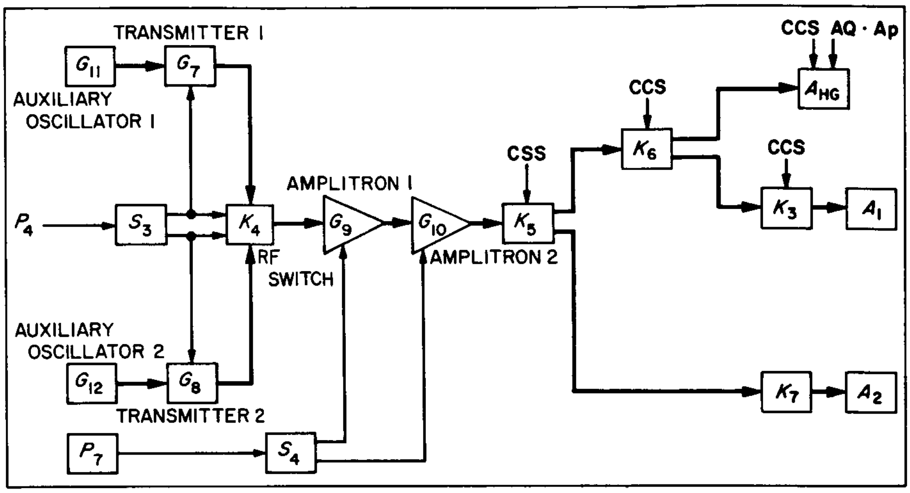


Fig. 26. Functional diagram for one-way doppler function

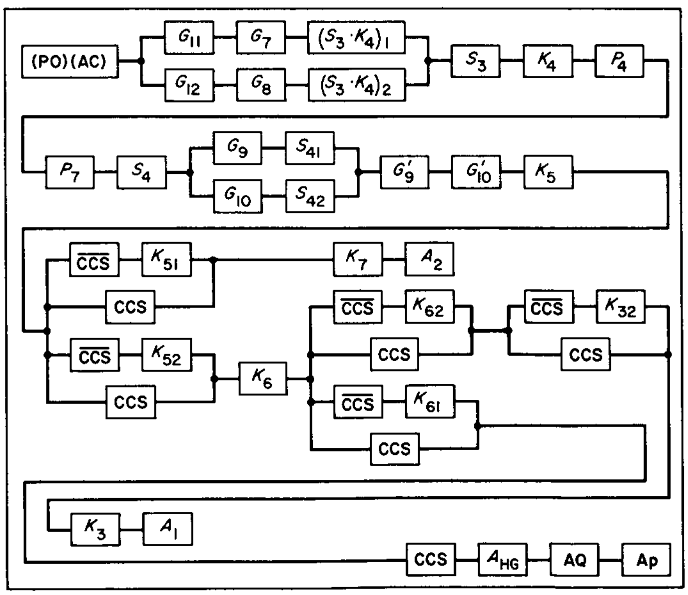


Fig. 27. Reliability diagram for one-way doppler function

The success event for this one-way doppler function D_O is given by

$$
\begin{aligned}
D_O = {} & (\mathbf{PO})\cdot(\mathbf{AC})\cdot K_4\cdot K_5\cdot P_4\cdot P_7\cdot S_3\cdot S_4\cdot [G_7\cdot G_{11}\cdot (S_3\cdot K_4)_1 + G_8\cdot G_{12}\cdot (S_3\cdot K_4)_2] \\
& \cdot\ [G_9\cdot S_{41} + G_{10}\cdot S_{42}]\cdot \{(\mathbf{CCS} + \overline{\mathbf{CCS}}\cdot K_{51})\cdot A_2\cdot K_7 + (\mathbf{CCS} + \overline{\mathbf{CCS}}\cdot K_{52})\cdot K_6 \\
& \cdot\ [(\mathbf{AQ})\cdot \mathbf{Ap}\cdot A_{HG}\cdot (\mathbf{CCS})\cdot(\mathbf{CCS} + \overline{\mathbf{CCS}}\cdot K_{61}) + A_1\cdot K_3\,(\mathbf{CCS} + \overline{\mathbf{CCS}}\cdot K_{31})(\mathbf{CCS} + \overline{\mathbf{CCS}}\cdot K_{62})]\}\cdot G_9'\cdot G_{10}' \\
= {} & (\mathbf{PO})\cdot(\mathbf{AC})\cdot K_4\cdot K_5\cdot P_4\cdot P_7\cdot S_3\cdot S_4\cdot [G_7\cdot G_{11}\cdot (S_3\cdot K_4)_1 + G_8\cdot G_{12}\cdot (S_3\cdot K_4)_2] \\
& \cdot\ [G_9\cdot S_{41} + G_{10}\cdot S_{42}]\cdot \{(\mathbf{CCS} + \overline{\mathbf{CCS}}\cdot K_{51})\, A_2\cdot K_7 + K_6\cdot [(\mathbf{AQ})\cdot \mathbf{Ap}\cdot A_{HG}\cdot (\mathbf{CCS}) \\
& \quad + A_1\cdot K_3\cdot (\mathbf{CCS} + \overline{\mathbf{CCS}}\cdot K_{32}\cdot K_{52}\cdot K_{62})]\}\cdot G_9'\cdot G_{10}' \\
= {} & (\mathbf{PO})\cdot(\mathbf{AC})\cdot K_4\cdot K_5\cdot P_4\cdot P_7\cdot S_3\cdot S_4\cdot [G_7\cdot G_{11}\,(S_3\cdot K_4)_1 + G_8\cdot G_{12}\cdot (S_3\cdot K_4)_2] \\
& \cdot\ [G_9\cdot S_{41} + G_{10}\cdot S_{42}]\cdot \{[A_2\cdot K_7 + (\mathbf{AQ})\cdot \mathbf{Ap}\cdot A_{HG}\cdot K_6 + A_1\cdot K_3\cdot K_6]\cdot \mathbf{CCS} \\
& + [A_2\cdot K_7\cdot K_{51} + A_1\cdot K_3\cdot K_{32}\cdot K_{52}\cdot K_6\cdot K_{62}]\cdot \overline{\mathbf{CCS}}\}\cdot G_9'\cdot G_{10}'
\end{aligned}
\tag{64}
$$

The derivation of the probability of success equation Pr $[D_O]$ is given in Appendix C. The equation Pr $[D_O]$ has a form

$$
\Pr\,[D_O] = h_1 + h_2 \Pr\,[\mathbf{CCS}] + h_3 \Pr\,[\mathbf{CCS}]\cdot \Pr\,[\mathbf{AQ}\cdot \mathbf{Ap}] \tag{65}
$$

which indicates that when Pr $[\mathbf{CCS}]$ is zero, the probability of Pr $[D_O]$ is independent of Pr $[\mathbf{AQ}\cdot\mathbf{Ap}]$ and is equal to h_1.

It is now possible to evaluate the one- or two-way doppler function. Since Pr $[D_O]$ and Pr $[D_T]$ are available, only the joint probability Pr $[D_O\cdot D_T]$ is required:

$$\Pr\,[D_O + D_T] = \Pr\,[D_O] + \Pr\,[D_T] - \Pr\,[D_O \cdot D_T] \tag{66}$$

The joint probability is derived in Appendix D.

G. Range-Tracking Function Reliability Analysis

There are two ranging methods used in the *Mariner B* spacecraft; namely, the turnaround ranging and the coded ranging. The former method is limited to a maximum range of approximately one million kilometers, which is a relatively short distance compared to the distances of planet encounters. For this analysis, only the coded ranging system will be considered.

Two-way communication is necessary for range tracking of a spacecraft. Subsequently, the reliability diagram for this function is similar to the two-way doppler tracking function. However, only one receiver, R_1, is equipped to operate with the ranging subsystem; the other receiver serves no purpose in this function and is not included in the reliability diagram.

A reliability diagram for this function is given in Fig. 28.

Using the same available signal paths as listed in the two-way doppler analysis, the reliability equation for the range-tracking function is immediately realized:

$$R_t = (\mathbf{PO}) \cdot (\mathbf{AC}) \cdot (\mathbf{RG}) \cdot P_4 \cdot K_2 \cdot R_1 \cdot (S_2 + S_{21}) \cdot [T] \cdot \{\mathrm{B}_1 \cdot \mathrm{C}_2 \cdot K_1 \cdot (\mathrm{L}_1 \cdot \mathrm{B}_5 + \mathrm{L}_2 \cdot \mathrm{B}_6)$$
$$+ \mathrm{C}_1 \cdot [\mathrm{L}_1 \cdot \mathrm{B}_1 \cdot \mathrm{B}_2 + \mathrm{L}_2 \cdot (\mathrm{B}_{62} + \mathrm{B}_2 \cdot \mathrm{B}_{61})]\} \tag{67}$$

where

R_t = success event of the range-tracking function

(RG) = success event of the ranging subsystem

All other events have been defined in previous Sections. Appendix E contains the complete derivation of the probability of success of this function.

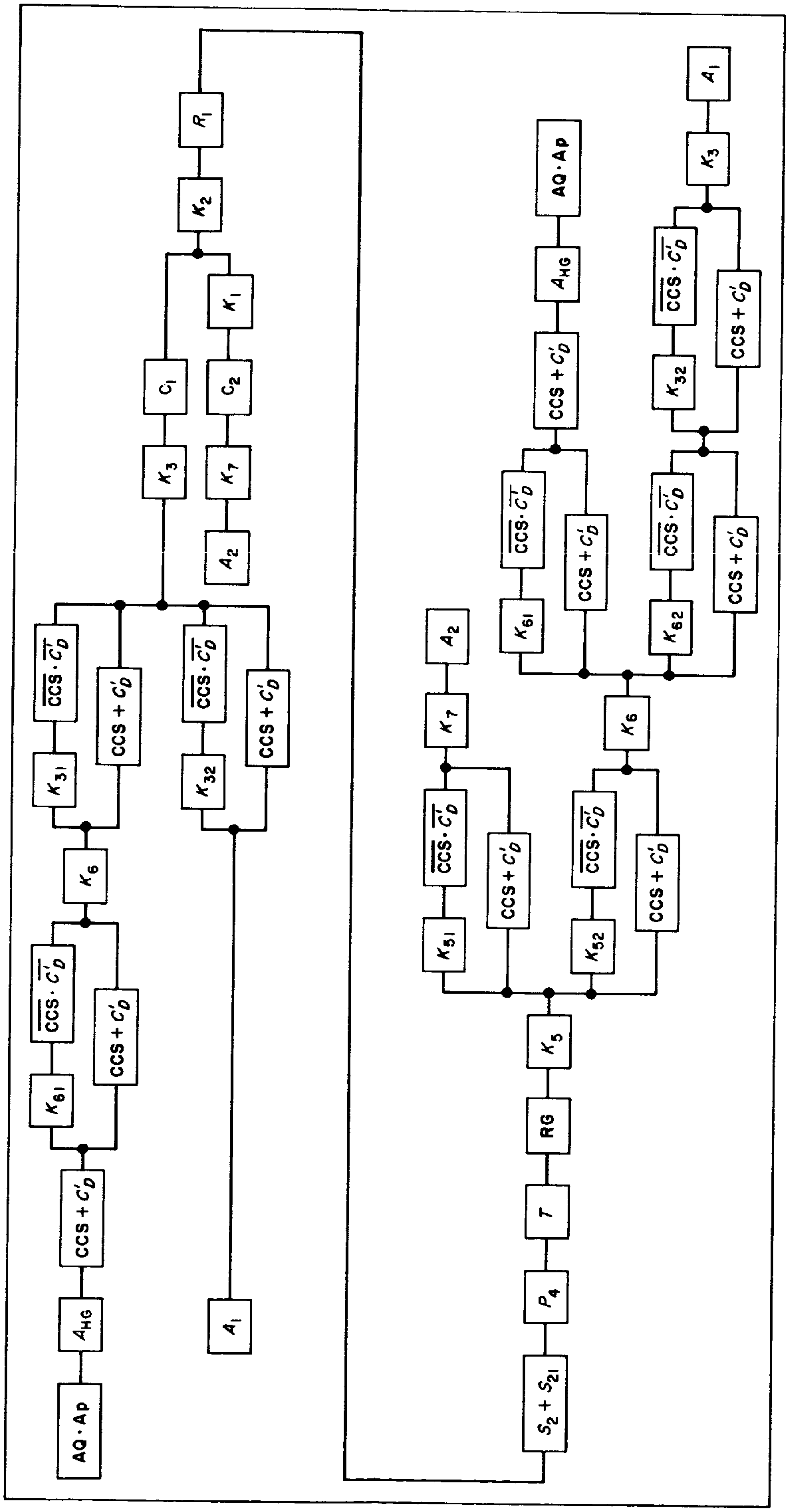


Fig. 28. Reliability diagram for range-tracking function

H. Telemetry Function Reliability Analysis

The prime objective of the *Mariner B* mission is to transmit scientific and engineering information to Earth during the interplanetary flight and at planet encounter. Consequently, the success of the telemetry function may be thought of as the success of the communication system. The encoded data modulates the RF carrier which may be generated by either the auxiliary crystal oscillator when there is no radio signal present at the receiver or the receiver voltage-controlled oscillator when the receiver is locked to a signal transmitted from the ground. Hence, the success event of the telemetry function is essentially the joint success event of the telemetry-encoding equipment and the one- or two-way communication function.

A simplified function reliability diagram is shown in Fig. 29.

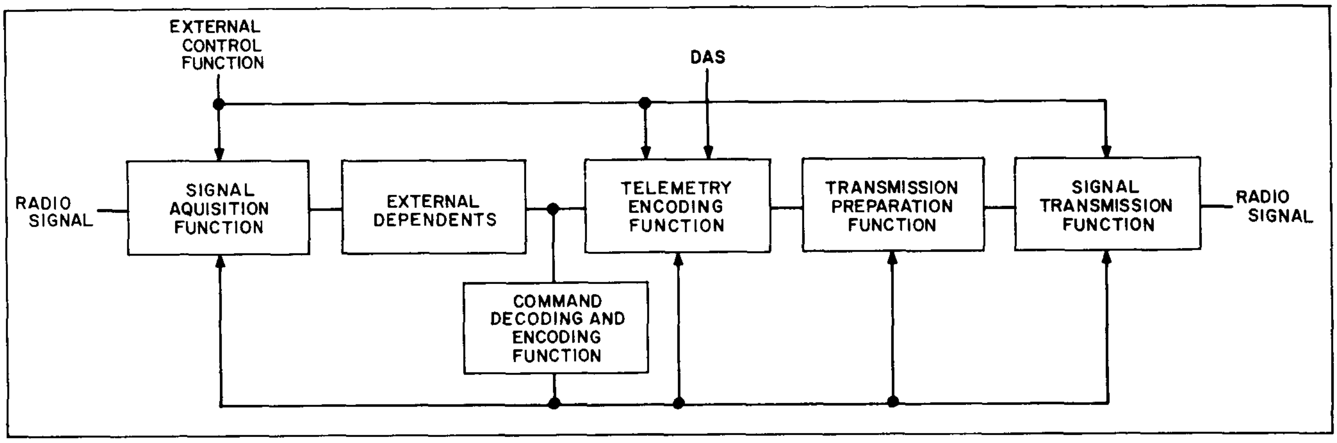


Fig. 29. Functional reliability diagram for telemetry function

With the exception of the addition of the telemetry-encoding function, this diagram is similar to the two-way doppler functional diagram which is given in Fig. 21. One can stipulate that the success event of the telemetry function is the product of the events of the telemetry-encoding function and the one- or two-way doppler function, as indicated in

$$TE = T_e \cdot (D_O + D_T) \tag{68}$$

where

TE = success event of telemetry function

T_e = success event of telemetry-encoding function

Success Event T_e

Figure 30 represents the functional block diagram for the telemetry-encoding function. Digital data are not included for simplicity. As can be seen, the telemetry subsystem is capable of accepting scientific data which are gathered in the Data Automation System (**DAS**), as well as from its own data encoder. It would be only fair to treat the two data sources separately before any reliability measure of this function is defined.

The command signals play a very important role in the success of the telemetry-encoding function. Proper switching from time to time, especially at planet encounter, must be accomplished. Therefore, ($\mathbf{CCS} + C_D'$) will be a series block in the reliability diagrams given in Fig. 31 and Fig. 32. Notice that these diagrams represent a two-way communication condition since the command subsystem is involved in the diagrams. However, when only one-way communication is available, C_D' is necessarily made zero because there will be no command from the ground available if the radio signal does not acquire the transponder receiver.

Equations (69) and (70) are the event equations for the telemetry-encoding function for engineering data and DAS data, respectively.

$$
\begin{aligned}
T_{e(\mathrm{ENGR})} = {} & \{N_1 \cdot [(\mathbf{CCS} + C_D') + \overline{\mathbf{CCS}} \cdot \overline{C_D'} \cdot N_{31}] + N_2 \cdot [(\mathbf{CCS} + C_D') + \overline{\mathbf{CCS}} \cdot \overline{C_D'} \cdot N_{32}]\} \\
& \cdot N_3 \cdot N_4 \cdot N_5 \cdot N_6 \cdot (\mathbf{CCS} + C_D') \cdot \{N_7 \cdot N_9 \cdot N_{11} \cdot N_{13} \cdot [(\mathbf{CCS} + C_D') + \overline{\mathbf{CCS}} \cdot \overline{C_D'} \cdot N_{61}] \\
& + N_8 \cdot N_{10} \cdot N_{12} \cdot N_{14} \cdot [(\mathbf{CCS} + C_D') + \overline{\mathbf{CCS}} \cdot \overline{C_D'} \cdot N_{62}]\} \cdot N_{15} \cdot N_{16} \cdot N_{17} \cdot N_{18} \cdot N_{19} \cdot N_{20} \cdot P_8 \\
= {} & (\mathbf{CCS} + C_D') \cdot [N_1 + N_2] \cdot N_3 \cdot N_4 \cdot N_5 \cdot N_6 \cdot [N_7 \cdot N_9 \cdot N_{11} \cdot N_{13} + N_8 \cdot N_{10} \cdot N_{12} \cdot N_{14}] \\
& \cdot N_{15} \cdot N_{16} \cdot N_{17} \cdot N_{18} \cdot N_{19} \cdot N_{20} \cdot P_8
\end{aligned} \tag{69}
$$

$$
\begin{aligned}
T_{e(\mathrm{DAS})} = {} & (\mathbf{CCS} + C_D') \cdot [N_1 + N_2] \cdot N_3 \cdot N_4 \cdot N_5 \cdot N_6 \cdot [N_7 \cdot N_9 \cdot N_{11} + N_8 \cdot N_{10} \cdot N_{12}] \\
& \cdot N_{15} \cdot N_{18} \cdot (\mathbf{DAS}) \cdot P_8
\end{aligned} \tag{70}
$$

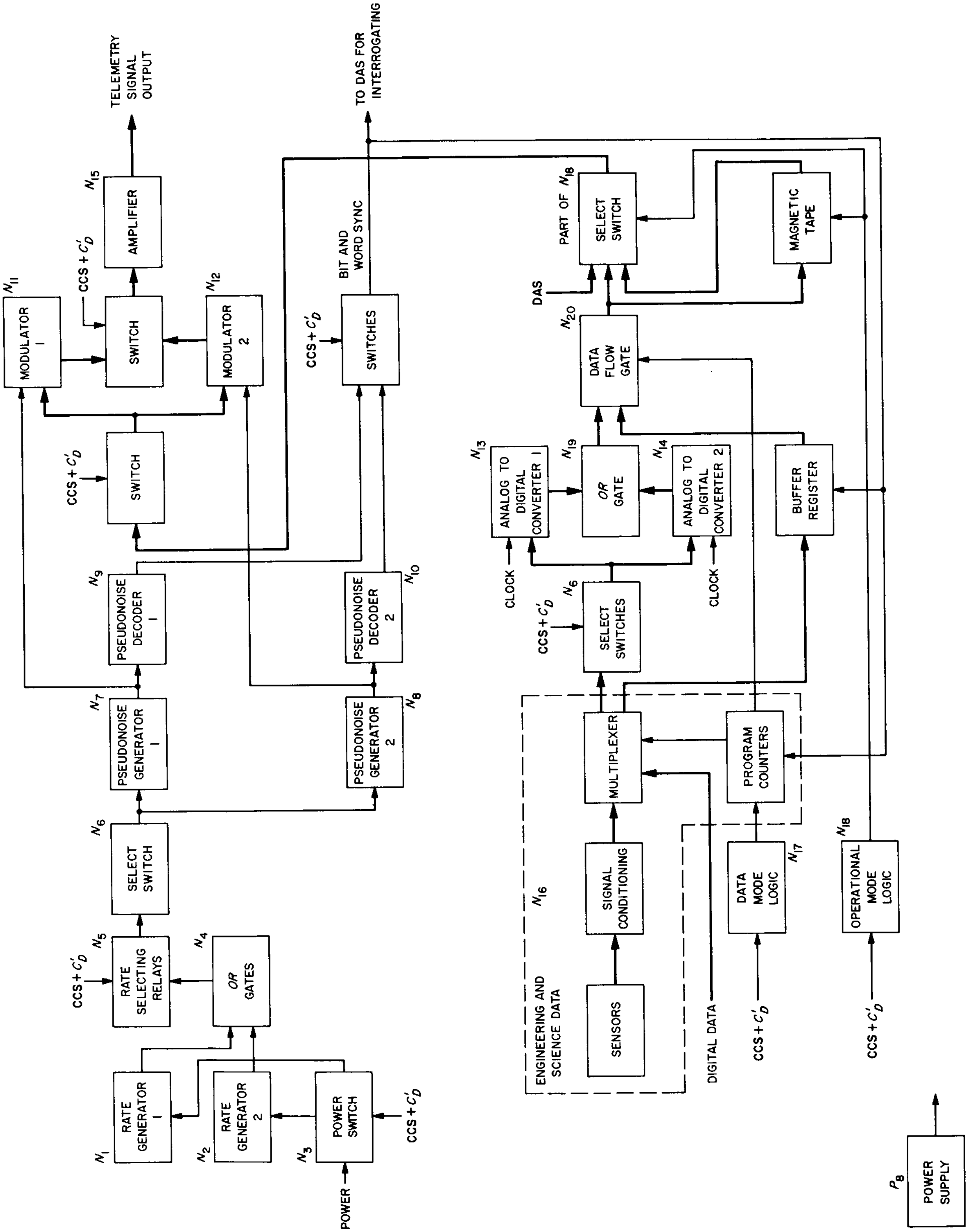


Fig. 30. Functional block diagram for telemetry-encoding function

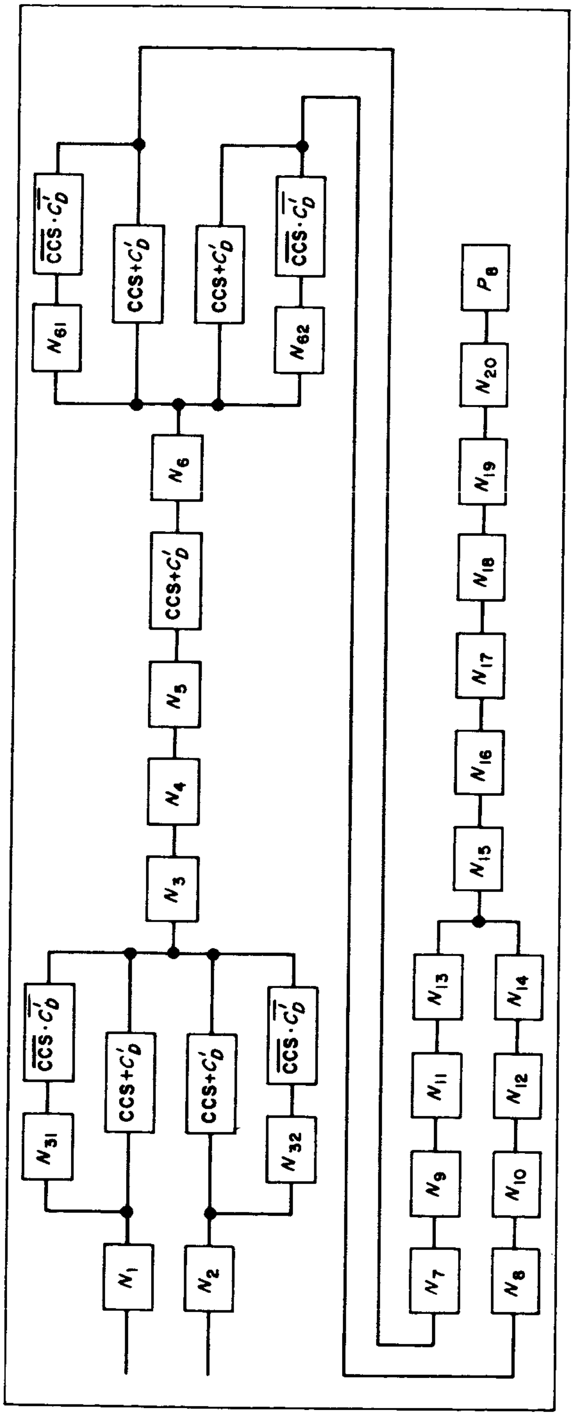


Fig. 31. Reliability diagram for telemetry-encoding function with engineering data

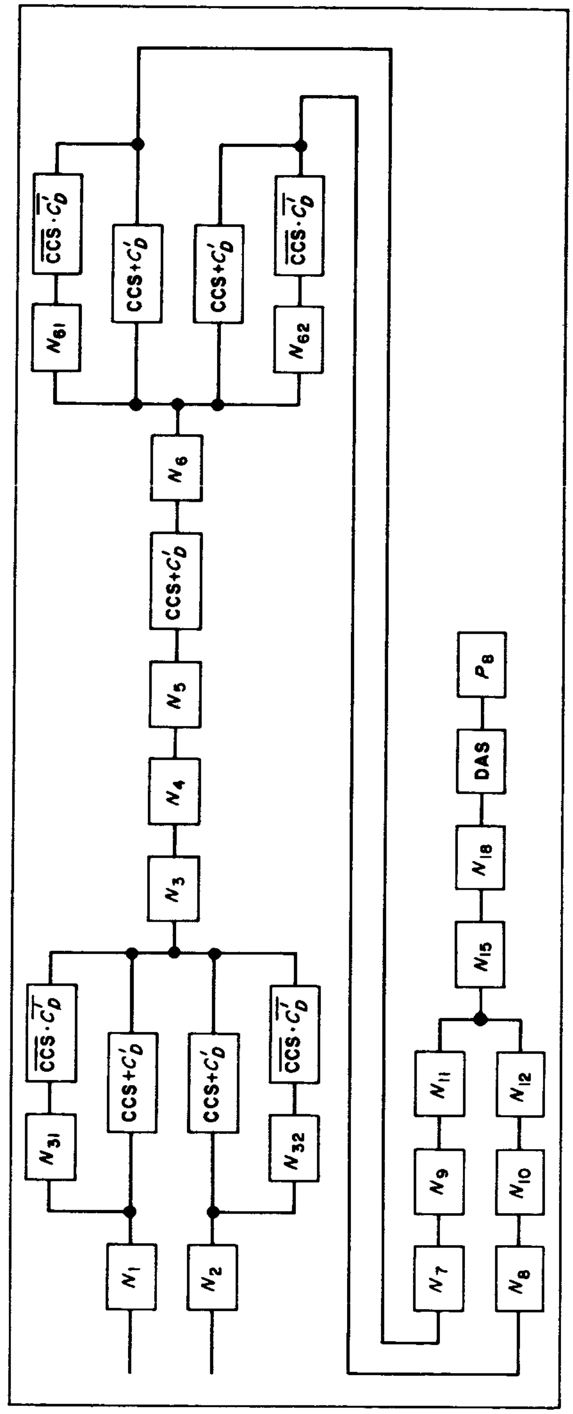


Fig. 32. Reliability diagram for telemetry-encoding function with DAS data

The partial success terms N_{3i} and N_{6i} disappear from these equations because of the importance of the command functions (**CCS**) and (C_D). The union and the intersection of these two events, $T_{e(\mathrm{ENGR})}$ and $T_{e(\mathrm{DAS})}$, are given in Eq. (71) and (72):

$$T_{e(\mathrm{ENGR})} + T_{e(\mathrm{DAS})} = (\mathbf{CCS} + C_D') \cdot [N_1 + N_2] \cdot N_3 \cdot N_4 \cdot N_5 \cdot N_6 \cdot N_{15} \cdot N_{18} \cdot P_8$$
$$\cdot \{ [N_7 \cdot N_9 \cdot N_{11} \cdot N_{13} + N_8 \cdot N_{10} \cdot N_{12} \cdot N_{14}] \cdot N_{16} \cdot N_{17} \cdot N_{19} \cdot N_{20}$$
$$+ [N_7 \cdot N_9 \cdot N_{11} + N_8 \cdot N_{10} \cdot N_{12}] \cdot (\mathbf{DAS}) \} \tag{71}$$

$$T_{e(\mathrm{ENGR})} \cdot T_{e(\mathrm{DAS})} = (\mathbf{CCS}) + C_D') \cdot [N_1 + N_2]\, N_3 \cdot N_4 \cdot N_5 \cdot N_6 \cdot P_8$$
$$\cdot [N_7 \cdot N_9 \cdot N_{11} \cdot N_{13} + N_8 \cdot N_{10} \cdot N_{12} \cdot N_{14}] \cdot N_{15} \cdot N_{16} \cdot N_{17} \cdot N_{18} \cdot N_{19} \cdot N_{20} \cdot \mathbf{DAS} \tag{72}$$

where

N_1 = success event of rate generator 1

N_2 = success event of rate generator 2

N_3 = success event of rate generator power switch

N_{31} = event in which rate generator 1 is energized when N_3 has failed

N_{32} = event in which rate generator 2 is energized when N_3 has failed

N_4 = success event of the *OR* gates for the rate generators'output

N_5 = success event of rate-selecting relay

N_6 = success event of selecting switches for PN generators

N_{61} = event in which PN generator 1 is energized when N_6 has failed

N_{62} = event in which PN generator 2 is energized when N_6 has failed

N_7 = success event of PN generator 1

N_8 = success event of PN generator 2

N_9 = success event of PN decoder 1

N_{10} = success event of PN decoder 2

N_{11} = success event of modulator 1

N_{12} = success event of modulator 2

N_{13} = success event of analog-to-digital converter 1

N_{14} = success event of analog-to-digital converter 2

N_{15} = success event of output amplifier

N_{16} = success event of multiplexing equipment

N_{17} = success event of data mode logic

N_{18} = success event of operational mode logic

N_{19} = success event of the OR gate for the analog-to-digital converters' output

N_{20} = success event of data flow gates

P_8 = success event of encoder power supply

Since the control term $(\mathbf{CCS} + C_D')$ is independent of the rest of the terms in Eq. (73) to (76), [assuming that the event $(\mathbf{DAS})$ occurs without inclusion of $(\mathbf{CCS})$], these equations may be written as

$$T_{e(\mathrm{ENGR})} = (\mathbf{CCS} + C_D') \cdot T'_{e(\mathrm{ENGR})} \tag{73}$$

$$T_{e(\mathrm{DAS})} = (\mathbf{CCS} + C_D') \cdot T'_{e(\mathrm{DAS})} \tag{74}$$

$$T_{e(\mathrm{ENGR})} \cdot T_{e(\mathrm{DAS})} = (\mathbf{CCS} + C_D') \cdot (T'_{e(\mathrm{ENGR})} \cdot T'_{e(\mathrm{DAS})}) \tag{75}$$

$$T_{e(\mathrm{ENGR})} + T_{e(\mathrm{DAS})} = (\mathbf{CCS} + C_D') \cdot (T'_{e(\mathrm{ENGR})} + T'_{e(\mathrm{DAS})}) \tag{76}$$

Hence

$$\begin{aligned}
\Pr\,[T'_{e(\mathrm{ENGR})} + T'_{e(\mathrm{DAS})}] = {} & \Pr\,[N_1 + N_2] \cdot \Pr\,[N_3 \cdot N_4 \cdot N_5 \cdot N_6 \cdot N_{15} \cdot N_{18}] \cdot \Pr\,[P_8] \\
& \cdot \{\Pr\,[N_7 \cdot N_9 \cdot N_{11} \cdot N_{13} + N_8 \cdot N_{10} \cdot N_{12} \cdot N_{14}] \cdot \Pr\,[N_{16} \cdot N_{17} \cdot N_{19} \cdot N_{20}] \\
& + \Pr\,[N_7 \cdot N_9 \cdot N_{11} + N_8 \cdot N_{10} \cdot N_{12}]\ \Pr\,[\mathbf{DAS}] - \Pr\,[N_7 \cdot N_9 \cdot N_{11} \cdot N_{13} \\
& + N_8 \cdot N_{10} \cdot N_{12} \cdot N_{14}] \cdot \Pr\,[N_{16} \cdot N_{17} \cdot N_{19} \cdot N_{20}] \cdot \Pr\,[\mathbf{DAS}]\,\}
\end{aligned}$$

$$\Pr\,[T'_{e(\mathrm{ENGR})} + T'_{e(\mathrm{DAS})}] = \Pr\,[N_1 + N_2] \cdot \Pr\,[N_3 \cdot N_4 \cdot N_5 \cdot N_6 \cdot N_{15} \cdot N_{18}] \cdot \Pr\,[P_8]$$

$$\cdot \{ \Pr\,[N_7 \cdot N_9 \cdot N_{11} \cdot N_{13} + N_8 \cdot N_{10} \cdot N_{12} \cdot N_{14}] \cdot \Pr\,[N_{16} \cdot N_{17} \cdot N_{19} \cdot N_{20}]$$

$$\cdot\ [1 - \Pr\,[\mathbf{DAS}]\,] + \Pr\,[N_7 \cdot N_9 \cdot N_{11} + N_8 \cdot N_{10} \cdot N_{12}] \cdot \Pr\,[\mathbf{DAS}]\} \tag{77}$$

The probabilities of success for $T'_{e(\mathrm{ENGR})}$, $T'_{e(\mathrm{DAS})}$ and $(T'_{e(\mathrm{ENGR})} \cdot T'_{e(\mathrm{DAS})})$ may be obtained in a similar manner. To obtain the telemetry function event equation, Eq. (78) may be adopted:

$$TE = T'_e \cdot (\mathbf{CCS} + C'_D) \cdot (D_O + D_T) = T'_e \cdot [(\mathbf{CCS}) \cdot D_O + (\mathbf{CCS} + C'_D) \cdot D_T] \tag{78}$$

The T_e may be selected from Eq. (73), (74), (75), or (76), depending upon the definition of the success parameter. Appendix F contains a step-by-step derivation of the probability of success for

$$(\mathbf{CCS}) \cdot D_O + (\mathbf{CCS} + C'_D) \cdot D_T \tag{79}$$

It should be noted that the multiplexing equipment, together with the sensors and signal-conditioning circuitry, is treated as an independent integral block. The reliability of this block is evaluated in Section II-B.

II. COMPUTATION OF FUNCTIONAL RELIABILITIES

A. General Considerations

After the reliability equations and their probabilistic counterparts of various defined functions in the *Mariner B* spacecraft communication system are developed, it is necessary to reduce them to some numerical representations from which meaningful illustrations may be derived. The quantitative results provide a clear visualization of the performance and offer a guide to improve the weaker links of the system from the reliability standpoint.

In order to obtain the probabilities of success for these functions, individual block reliabilities must be first evaluated. Each block consists of a number of circuits and components essential to the successful operation of that block. To obtain a reasonable reliability figure for a block, the exact number of the various types of essential components, together with their acceptable failure rate estimates, must be known. In short, complete documentation for the detailed design and the testing data of the various types of components used within the system must be available. In this study, detailed design records for the spacecraft command and data encoder subsystems are available. These records permit an accurate parts count of the blocks associated with them. Less fortunate is the radio subsystem blocks evaluation, as no detailed data are available. The figures adopted for these blocks are based on comparable equipment evaluated from the *Mariner A* communication system reliability study project. Most of the component failure rates are selected to be the same as those used in the above-mentioned study. Although absolute reliability prediction may not be achieved, it offers an excellent means for the comparison of the relative performance between the two systems and establishes a reference for future reliability improvement of spacecraft communication system design.

Several assumptions are made in obtaining the individual block reliability figures. These assumptions are listed here.

1. Good engineering practice is applied to all circuit designs so that circuit failures are confined to catastrophic failure or drastic degradation of components.
2. Careful quality control procedure is assured to eliminate all the early failures of components.
3. Part failures are random in time with constant failure rates over the entire course of mission.

Based on these assumptions, a first-order differential equation that describes the component failure characteristics can be established and is given by

$$\frac{dN}{dt} = -\lambda N \tag{80}$$

where

N = number of good components of the same kind at any time

λ = constant failure rate of components of the same kind

Applying the variables separable method yields

$$N = e^{-\lambda + c} = e^{c} \cdot e^{-\lambda t} \tag{81}$$

Here e^c is the initial number of good components and may be denoted as N_0.

From the very basic consideration of reliability concepts, the reliability of a component may be defined as the ratio of N and N_0 at any time. Hence

$$\text{Reliability} = \frac{N}{N_0} = e^{-\lambda t} \tag{82}$$

Equation (82) is generally known as the exponential failure law of a component.

In addition to the above assumptions, it is further assumed that the time durations of the space flights are 2,000 hr and 4,000 hr for the Venus mission and the Mars mission, respectively. A stress factor of two is assigned to all analog-type circuits to compensate for the stricter tolerance requirements. A stress factor of one is adopted for all digital circuitry. These assumptions conform to the *Mariner A* study effort. Block reliability is then computed according to the following equation:

$$R_B = e^{-\sum_{i=1}^{n} S_i \lambda_i t} \tag{83}$$

where

S = stress factor

t = mission duration

λ = component failure rate

n = total number of essential parts within a block

Equation (83) is applicable to most blocks, the only exception being that of the multiplexing equipment of the telemetry subsystem. A special technique is developed for the calculation of the multiplexer reliability, and it will be illustrated in Section II-B.

Block reliabilities of the *Mariner B* spacecraft communication system are tabulated in Table 2.

Table 2. Block reliability tabulation

Functional block success event		Reliability figure	
Symbol	**Definition**	**Venus distance**	**Mars distance**
P_1	Command detector transformer-rectifier unit 1	0.9894	0.9789
P_2	Command detector transformer-rectifier unit 2	0.9894	0.9789
P_4	Radio subsystem transformer-rectifier unit	0.9000	0.8100
P_7	Amplitron power supply	0.998	0.997
P_8	Data encoder power supply	0.890	0.792
G_1	Command input amplifier	1.000[a]	1.000[a]
G_2	Command detector 1	0.7325	0.537
G_3	Command detector 2	0.7325	0.537
G_4	Power selector	0.9885	0.9772
G_5	Decoder access switch	0.9896	0.9792
G_6	Command decoder	0.8408	0.707
G_7	Transmitter 1	0.945	0.893
G_8	Transmitter 2	0.945	0.893

[a] A value of one is assigned for this block because G_1 does not physically exist.

Table 2 (Cont'd)

Functional block success event		Reliability figure	
Symbol	Definition	Venus distance	Mars distance
G_9	Amplitron 1 (amplifying function)	0.986	0.974
G_{10}	Amplitron 2 (amplifying function)	0.986	0.974
G_9'	Amplitron 1 (transmission line function)	0.999	0.998
G_{10}'	Amplitron 2 (transmission line function)	0.999	0.998
G_{11}	Auxiliary oscillator 1	0.971	0.945
G_{12}	Auxiliary oscillator 2	0.971	0.945
A_1	Low-gain antenna 1	0.968	0.937
A_2	Low-gain antenna 2 (preferred)	0.968	0.937
A_{HG}	High-gain antenna	0.930	0.866
R_1	Receiver 1	0.789	0.625
R_2	Receiver 2	0.789	0.625
S_1	Command detector selector	0.9971	0.9942
S_{11}	Partial success of S_1	0.0006	0.0012
S_{12}	Partial success of S_1	0.0006	0.0012
S_2	Receiver output selector	0.9856	0.9716
S_{21}	Partial success of S_2	0.0008	0.0011
S_{22}	Partial success of S_2	0.0008	0.0011
S_3	Transmitter power-selecting switch	0.990	0.980
$(S_3 \cdot K_4)_1$	Partial success of S_3 and K_4	0.495	0.495
$(S_3 \cdot K_4)_2$	Partial success of S_3 and K_4	0.495	0.495
S_4	Amplitron power-selecting switch	0.990	0.980
S_{41}	Partial success of S_4	0.5	0.5
S_{42}	Partial success of S_4	0.5	0.5
K_1	RF switch	0.996	0.992
K_2	RF switch	0.996	0.992
K_3	RF switch	0.996	0.992

Table 2 (Cont'd)

Functional block success event		Reliability figure	
Symbol	Definition	Venus distance	Mars distance
K_{31}	Partial success of K_3	0.5	0.5
K_{32}	Partial success of K_3	0.5	0.5
K_4	RF switch	0.996	0.992
K_5	RF switch	0.996	0.992
K_{51}	Partial success of K_5	0.5	0.5
K_{52}	Partial success of K_5	0.5	0.5
K_6	RF switch	0.996	0.992
K_{61}	Partial success of K_6	0.5	0.5
K_{62}	Partial success of K_6	0.5	0.5
K_7	RF switch	0.996	0.992
M	Memory element (cross-switching configuration)	0.9971	0.9942
M_{11}	Partial success of M	0.0006	0.0012
M_{12}	Partial success of M	0.0006	0.0012
M_{21}	Partial success of M	0.5	0.5
M_{22}	Partial success of M	0.5	0.5
SK	Switch drive for command function only	0.9844	0.9691
SK_1'	Switch drive for other functions	0.9624	0.9262
N_1	Rate generator 1	0.9758	0.9522
N_2	Rate generator 2	0.9758	0.9522
N_3	Rate generator power switch	0.9845	0.9692
N_4	Rate output *OR* gates	0.9925	0.9853
N_5	Rate-selecting relays	0.9405	0.8844
N_6	Selecting switches	0.9734	0.9475
N_7	PN generator 1	0.9870	0.9741
N_8	PN generator 2	0.9870	0.9741
N_9	PN decoder 1	0.9917	0.9834

Table 2 (Cont'd)

Functional block success event		Reliability figure	
Symbol	Definition	Venus distance	Mars distance
N_{10}	PN decoder 2	0.9917	0.9834
N_{11}	Modulator 1	0.9955	0.9911
N_{12}	Modulator 2	0.9955	0.9911
N_{13}	Analog-to-digital converter 1	0.931	0.8668
N_{14}	Analog-to-digital converter 2	0.931	0.8668
N_{15}	Output amplifier	0.995	0.991
N_{16}	Multiplexing equipment	0.825	0.679
N_{17}	Data mode logic	0.9817	0.9637
N_{18}	Operational mode logic	0.9345	0.8733
N_{19}	Analog-to-digital output *OR* gate	0.999	0.9985
N_{20}	Data flow gate	0.997	0.994

B. Multiplexing Equipment

Because of the complicated nature of the multiplexing equipment in the reliability sense, direct application of Eq. (83) to evaluate the probability of success of a multiplexer is not practical. In a data acquisition system, a multiplexer is used to direct a large number of input signals into a common data conversion unit in such a manner that only one input signal will be accepted at a time. The various input signals are accepted or rejected by means of controlling contact closures of a large number of switches which are interconnected in a predetermined fashion. In a multideck configuration, failures of different switches do not necessarily bear a similar degree of significance. A switch failure occurring in the primary deck essentially wipes out the entire group of signal inputs associated with that switch, while a switch failure occurring in the secondary or tertiary deck may cause only the loss of a single input signal. It is not justifiable to claim that the multiplexer has failed on the basis of a small percentage loss of the total input signals. The result of such loss degradates the effectiveness of the multiplexer; nevertheless, it is still useful in a limited sense.

To analyze this type of equipment, it is best to adapt a piece-wise approach. To compute the reliability figure, one begins with the lowest rank decks and gradually incorporates the higher rank elements until the entire unit has been included. Sensor, signal-conditioning circuit (if any), switch, and switch driver within a channel are lumped together as an integral part. By observing Fig. 33(c), one may find that a channel indeed is composed of these four components. The failure of any of these components causes the channel operation to fail.

To illustrate this method, a fictitous multiplexer with associated sensors and signal-conditioning equipment is devised. This fictitious unit has a similar construction to the one used in the *Mariner B* data encoder. It is a balanced-type commutation machine with three-level subcommutating decks. Each deck consists of five input channels, and the output of each deck may or may not be associated with a signal-conditioning device. Transducers are attached to each of the tertiary deck inputs. Each of these decks is driven by its own sequencer so that a sequence failure of a deck does not affect the proper sequencing of others. A simplified schematic diagram for this machine is shown in Fig. 33(b).

The first attempt is to look into the probability of success for each five-channel commutating deck. It should be noted that all inputs are equally important for this analysis to be valid. Furthermore, a commutator deck is considered a success provided that at least M channels are operating properly. In this case, M is a number equal to or less than five and is to be defined, depending on the degree of significance of the deck.

A truth table (Table 3) can immediately be established,representing all the combinations of failure or success of the five channels. From this truth table, the probability of exactly five channels operating successfully is

$$P_5 = R_{C1} \cdot R_{C2} \cdot R_{C3} \cdot R_{C4} \cdot R_{C5} \tag{84}$$

Similarly, the probabilities of exactly n channels operating successfully (n = 4, 3, 2, 1, and 0) are represented by Eq. (85) to (89),respectively.

$$\begin{aligned} P_4 = {} & R_{C1} \cdot R_{C2} \cdot R_{C3} \cdot R_{C4} \cdot (1 - R_{C5}) + R_{C1} \cdot R_{C2} \cdot R_{C3} \cdot R_{C5} \cdot (1 - R_{C4}) \\ & + R_{C1} \cdot R_{C2} \cdot R_{C4} \cdot R_{C5} \cdot (1 - R_{C3}) + R_{C1} \cdot R_{C3} \cdot R_{C4} \cdot R_{C5} \cdot (1 - R_{C2}) \\ & + R_{C2} \cdot R_{C3} \cdot R_{C4} \cdot R_{C5} \cdot (1 - R_{C1}) \end{aligned} \tag{85}$$

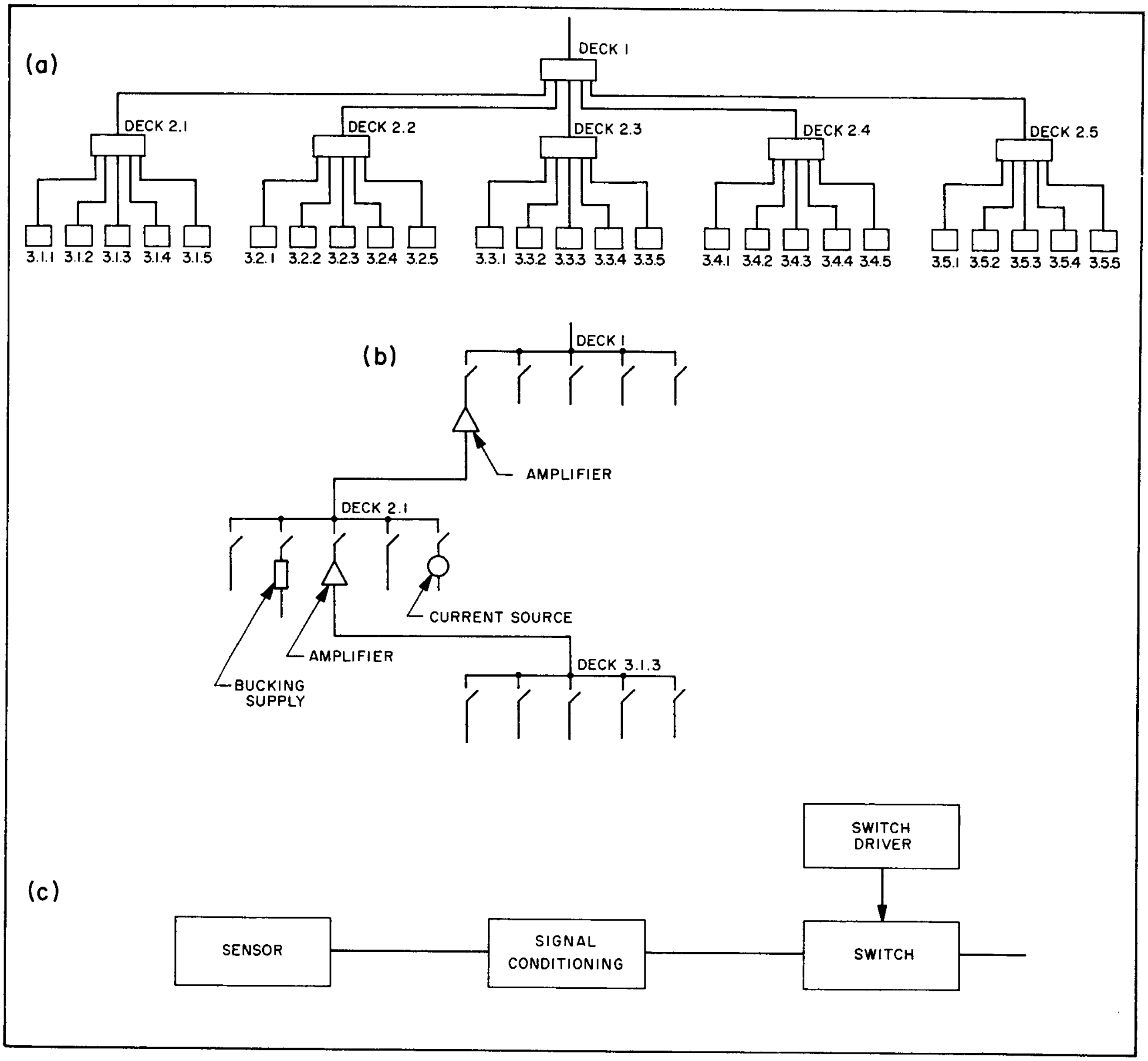


Fig. 33. Three-level five-channel multiplexer
a. Logical diagram
b. Partial schematic
c. Signal channel

$$
\begin{aligned}
P_3 = {} & R_{C1} \cdot R_{C2} \cdot R_{C3} \cdot (1 - R_{C4}) \cdot (1 - R_{C5}) + R_{C1} \cdot R_{C2} \cdot R_{C4} \cdot (1 - R_{C3}) \cdot (1 - R_{C5}) \\
& + R_{C1} \cdot R_{C3} \cdot R_{C4} \cdot (1 - R_{C2}) \cdot (1 - R_{C5}) + R_{C2} \cdot R_{C3} \cdot R_{C4} \cdot (1 - R_{C1}) \cdot (1 - R_{C5}) \\
& + R_{C1} \cdot R_{C2} \cdot R_{C5} \cdot (1 - R_{C3}) \cdot (1 - R_{C4}) + R_{C1} \cdot R_{C3} \cdot R_{C5} \cdot (1 - R_{C2}) \cdot (1 - R_{C4}) \\
& + R_{C2} \cdot R_{C3} \cdot R_{C5} \cdot (1 - R_{C1}) \cdot (1 - R_{C4}) + R_{C1} \cdot R_{C4} \cdot R_{C5} \cdot (1 - R_{C2}) \cdot (1 - R_{C3}) \\
& + R_{C2} \cdot R_{C4} \cdot R_{C5} \cdot (1 - R_{C1}) \cdot (1 - R_{C3}) + R_{C3} \cdot R_{C4} \cdot R_{C5} \cdot (1 - R_{C1}) \cdot (1 - R_{C2})
\end{aligned}
\tag{86}
$$

$$
\begin{aligned}
P_2 = {} & R_{C1} \cdot R_{C2} \cdot (1 - R_{C3}) \cdot (1 - R_{C4}) \cdot (1 - R_{C5}) + R_{C1} \cdot R_{C3} \cdot (1 - R_{C2}) \cdot (1 - R_{C4}) \cdot (1 - R_{C5}) \\
& + R_{C2} \cdot R_{C3} \cdot (1 - R_{C1}) \cdot (1 - R_{C4}) \cdot (1 - R_{C5}) + R_{C1} \cdot R_{C4} \cdot (1 - R_{C2}) \cdot (1 - R_{C3}) \cdot (1 - R_{C5}) \\
& + R_{C2} \cdot R_{C4} \cdot (1 - R_{C1}) \cdot (1 - R_{C3}) \cdot (1 - R_{C5}) + R_{C3} \cdot R_{C4} \cdot (1 - R_{C1}) \cdot (1 - R_{C2}) \cdot (1 - R_{C5}) \\
& + R_{C1} \cdot R_{C5} \cdot (1 - R_{C2}) \cdot (1 - R_{C3}) \cdot (1 - R_{C4}) + R_{C2} \cdot R_{C5} \cdot (1 - R_{C1}) \cdot (1 - R_{C3}) \cdot (1 - R_{C4}) \\
& + R_{C3} \cdot R_{C5} \cdot (1 - R_{C1}) \cdot (1 - R_{C2}) \cdot (1 - R_{C4}) + R_{C4} \cdot R_{C5} \cdot (1 - R_{C1}) \cdot (1 - R_{C2}) \cdot (1 - R_{C3})
\end{aligned}
\tag{87}
$$

$$
\begin{aligned}
P_1 = {} & R_{C1} \cdot (1 - R_{C2}) \cdot (1 - R_{C3}) \cdot (1 - R_{C4}) \cdot (1 - R_{C5}) + R_{C2} \cdot (1 - R_{C1}) \cdot (1 - R_{C3}) \cdot (1 - R_{C4}) \cdot (1 - R_{C5}) \\
& + R_{C3} \cdot (1 - R_{C1}) \cdot (1 - R_{C2}) \cdot (1 - R_{C4}) \cdot (1 - R_{C5}) + R_{C4} \cdot (1 - R_{C1}) \cdot (1 - R_{C2}) \cdot (1 - R_{C3}) \cdot (1 - R_{C5}) \\
& + R_{C5} \cdot (1 - R_{C1}) \cdot (1 - R_{C2}) \cdot (1 - R_{C3}) \cdot (1 - R_{C4})
\end{aligned}
\tag{88}
$$

$$
P_0 = (1 - R_{C1}) \cdot (1 - R_{C2}) \cdot (1 - R_{C3}) \cdot (1 - R_{C4}) \cdot (1 - R_{C5}) \tag{89}
$$

Table 3. Truth table of all possible combinations of channel failure or success for a five-channel commutation deck

R_{C1}	R_{C2}	R_{C3}	R_{C4}	R_{C5}
0	0	0	0	0
1	0	0	0	0
0	1	0	0	0
1	1	0	0	0
0	0	1	0	0
1	0	1	0	0
0	1	1	0	0
1	1	1	0	0
0	0	0	1	0
1	0	0	1	0
0	1	0	1	0
1	1	0	1	0
0	0	1	1	0
1	0	1	1	0
0	1	1	1	0
1	1	1	1	0
0	0	0	0	1
1	0	0	0	1
0	1	0	0	1
1	1	0	0	1
0	0	1	0	1
1	0	1	0	1
0	1	1	0	1
1	1	1	0	1
0	0	0	1	1
1	0	0	1	1
0	1	0	1	1
1	1	0	1	1
0	0	1	1	1
1	0	1	1	1
0	1	1	1	1
1	1	1	1	1

1 represents channel success
0 represents channel failure
R_i represents probability of success of channel i

The simplest case is that all the channel reliabilities are identical and are equal to R_C. Under this condition, Eq. (84) through (89) reduce to

$$P_5' = R_C^5 \tag{90}$$

$$P_4' = \binom{5}{4} R_C^4 \cdot (1 - R_C) \tag{91}$$

$$P_3' = \binom{5}{3} R_C^3 \cdot (1 - R_C)^2 \tag{92}$$

$$P_2' = \binom{5}{2} R_C^2 \cdot (1 - R_C)^3 \tag{93}$$

$$P_1' = \binom{5}{1} R_C \cdot (1 - R_C)^4 \tag{94}$$

$$P_0' = (1 - R_C)^5 \tag{95}$$

Furthermore, the probabilities for at least n channels operating successfully (n = 4, 3, 2, 1) are given by

$$P_4'' = P_4' + P_5' \tag{96}$$

$$P_3'' = P_3' + P_4' + P_5' \tag{97}$$

$$P_2'' = P_2' + P_3' + P_4' + P_5' \tag{98}$$

$$P_1'' = P_1' + P_2' + P_3' + P_4' + P_5' \tag{99}$$

A family of curves of P_i'' as functions of R_C is given in Fig. 34. This graph may be used as a quick reference for evaluating deck reliability if the per-channel reliability of each channel is the same.

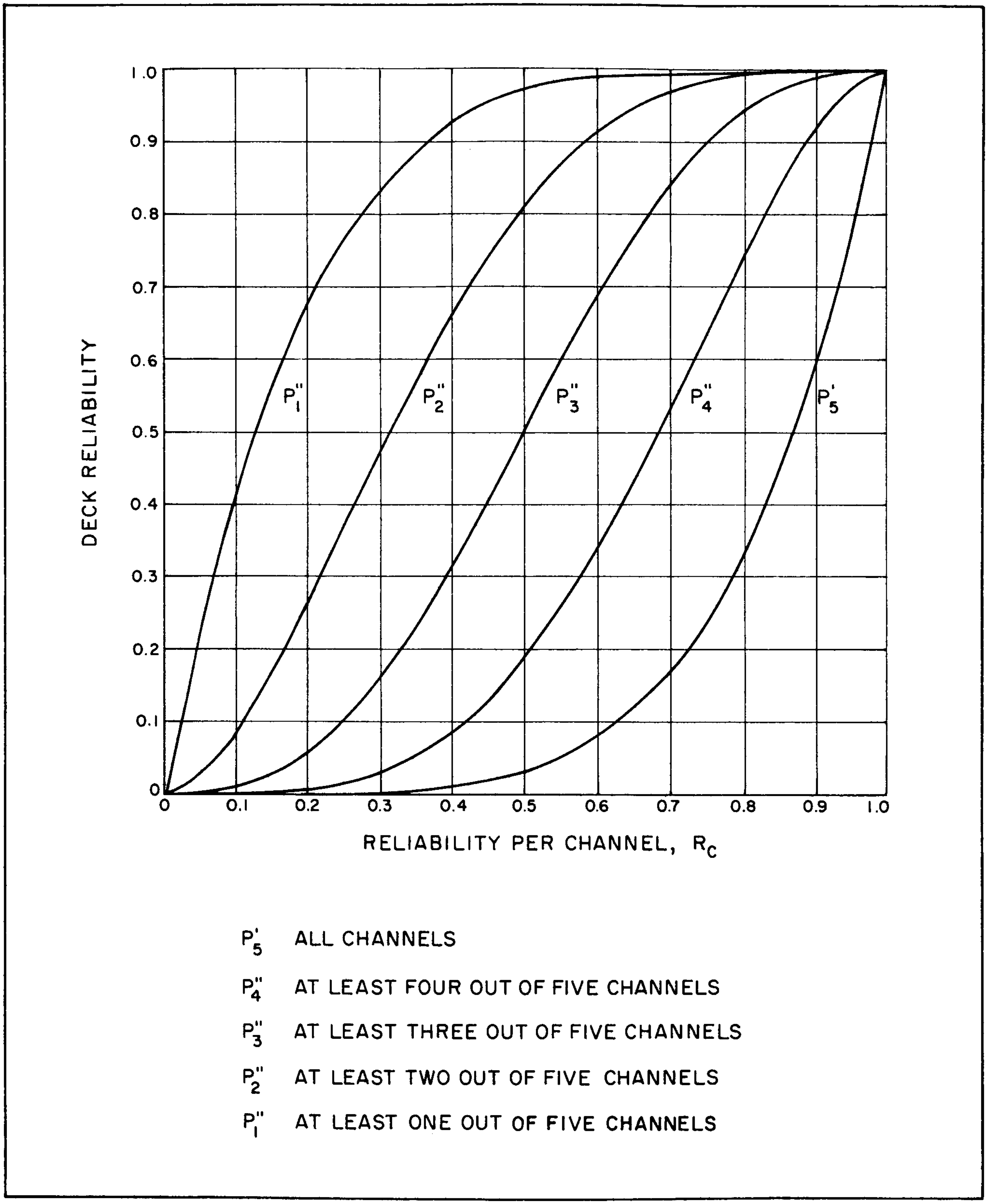


Fig. 34. Five-channel commutator deck

With these tools available, the evaluation of the multiplexing equipment (Fig. 33) reliability becomes straightforward. It will be illustrated by the following example. Let us assume that the reliabilities of the various circuits are as follows:

$$R_{sensor} = 1$$

$$R_{amp} = 0.95$$

$$R_{BS} = 0.97$$

$$R_{I} = 0.96$$

$$R_{SW} = 0.95$$

$$R_{SQ} = 0.99$$

Furthermore, $M = 4$ for all the second- and third-level commutator decks, and $M = 5$ for the primary deck. This assumption allows a maximum loss of 45 out of a total of 125 signal inputs for this machine's success.

Starting from the third-level decks, for example deck 3.1.3, since all the channel reliabilities are identical and are equal to 0.95, the deck reliability, excluding the reliability of the sequencer, is found to be 0.975 from Fig. 34. This figure is multiplied by 0.99, the sequencer reliability, to obtain the total reliability of the deck 3.1.3. With this figure obtained, the entire deck is assumed to be the transducer of the third channel of deck 2.1. Similar methods may be applied to other third-level decks. For deck 2.1, the per-channel reliability is not the same, and the use of Eq. (84) and (85) is required to obtain its deck reliability. This procedure is continued until the reliability of deck 1 has been obtained.

It can be seen that this method is more reasonable since it has considered the weighting of each switch. However, it becomes quite tedious to use if the number of channels per deck becomes larger and if all the channel reliabilities are different. The latter is generally not the case since, for a real system, many channels would bear the same configuration, and the result can be easily obtained by applying combinatorial mathematics.

The computation of the data encoder multiplexing equipment follows the above outline quite closely. However, the success event of each deck is defined separately, depending on its merit. Failure modes of switches are also included in the calculation.

C. Performance Margin of Low-Gain Antennas

The component reliability equation given in Eq. (82) neglects the incapabilities of system components. It is assumed that all components are within their normal life of operation for both the Venus and the Mars missions with no wear-out effects and are functioning regardless of the location relative to the Earth. Such a gross assumption is not valid for the low-gain antennas. It is realized that as the spacecraft is moving away from the Earth, there is a critical distance such that the signal strength radiated through the low-gain antennas falls below the required signal level for satisfactory reception. Beyond this critical point, the failure of the low-gain antennas becomes a certainty. Although the exact point has not been released, it is undoubtedly within the Mars distance. In evaluating the functional probabilities of success, a probability of zero should be assigned to both the low-gain antennas as soon as the spacecraft has reached this point.

To illustrate the effect on the reliability of the spacecraft communication system due to the sudden disability of the low-gain antennas, the command function reliability is plotted as a function of time. For the lack of a better number, the critical distance is arbitrarily chosen to be 3,000 hr of space flight. This curve is shown in Fig. 35. As can be expected, the curve is very close to an exponential one in time until the 3,000-hr point is reached. At that time, a sudden break takes place, followed by a gradual decay in an exponential fashion. The sudden drop in reliability is, of course, due to the loss of services from the two low-gain antennas and completely eliminates the redundancy of radio linkage between the spacecraft and the Earth's stations.

If the same calculation is made for the other spacecraft functions, similar characteristics can be expected.

D. Graphical Illustrations of Functional Successes in Terms of External Dependents

A collection of three-dimensional representations (Fig. 36-44) is enclosed for rapid reference. Functional successes are plotted with respect to both the Central Computer and Sequencer and the Canopus acquisition and antenna positioning probabilities of success. Equal-distance grid lines are also provided so that with a pair of given probabilities of the (**CCS**) and (**AQ** · **Ap**), the function success can be immediately measured.

These graphs may be categorized into two groups. The first group is for the Venus mission, while the other is for the Mars mission. It is noticed that function successes in the Venus mission are relatively unaffected by the Canopus acquisition and the antenna positioning function because of the highly redundant arrangement of the spacecraft antennas. On the other hand, the same function becomes of prime importance for the Mars mission because of the disability of the low-gain antennas at planet encounter. This is not to speculate that low-gain antennas are unnecessary for the Mars shot. Undeniably they contribute significant reliability improvements over the single antenna system during the earlier portion of the space flight.

The equations expressing these graphs are given as follows:

for Venus,

$$\Pr[C] = 0.6390 + 0.0011 \Pr[\mathbf{CCS}] + 0.0007 \Pr[\mathbf{AQ} \cdot \mathbf{Ap}] \tag{100}$$

$$\Pr[D_T] = 0.7101 + 0.0886 \Pr[\mathbf{CCS}] + 0.0007 \Pr[\mathbf{AQ} \cdot \mathbf{Ap}] + 0.0002 \Pr[\mathbf{CCS}] \cdot \Pr[\mathbf{AQ} \cdot \mathbf{Ap}] \tag{101}$$

$$\Pr[D_O] = 0.4695 + 0.3092 \Pr[\mathbf{CCS}] + 0.0009 \Pr[\mathbf{CCS}] \cdot \Pr[\mathbf{AQ} \cdot \mathbf{Ap}] \tag{102}$$

$$\Pr[D_T + D_O] = 0.7469 + 0.0983 \Pr[\mathbf{CCS}] + 0.0010 \Pr[\mathbf{AQ} \cdot \mathbf{Ap}] + 0.0003 \Pr[\mathbf{CCS}] \cdot \Pr[\mathbf{AQ} \cdot \mathbf{Ap}] \tag{103}$$

$$\Pr[TE] = 0.3519 + 0.1471 \Pr[\mathbf{CCS}] + 0.0004 \Pr[\mathbf{AQ} \cdot \mathbf{Ap}] \tag{104}$$

and for Mars,

$$\Pr[C] = 0.2890 \Pr[\mathbf{AQ} \cdot \mathbf{Ap}] + 0.0092 \Pr[\mathbf{CCS}] \cdot \Pr[\mathbf{AQ} \cdot \mathbf{Ap}] \tag{105}$$

$$\Pr[D_T] = 0.2558 \Pr[\mathbf{AQ} \cdot \mathbf{Ap}] + 0.2365 \Pr[\mathbf{CCS}] \cdot \Pr[\mathbf{AQ} \cdot \mathbf{Ap}] \tag{106}$$

$$\Pr[D_O] = 0.5314 \Pr[\mathbf{CCS}] \cdot \Pr[\mathbf{AQ} \cdot \mathbf{Ap}] \tag{107}$$

$$\Pr[D_T + D_O] = 0.2558 \Pr[\mathbf{AQ} \cdot \mathbf{Ap}] + 0.3337 \Pr[\mathbf{CCS}] \cdot \Pr[\mathbf{AQ} \cdot \mathbf{Ap}] \tag{108}$$

$$\Pr[TE] = 0.0881 \Pr[\mathbf{AQ} \cdot \mathbf{Ap}] + 0.1149 \Pr[\mathbf{CCS}] \cdot \Pr[\mathbf{AQ} \cdot \mathbf{Ap}] \tag{109}$$

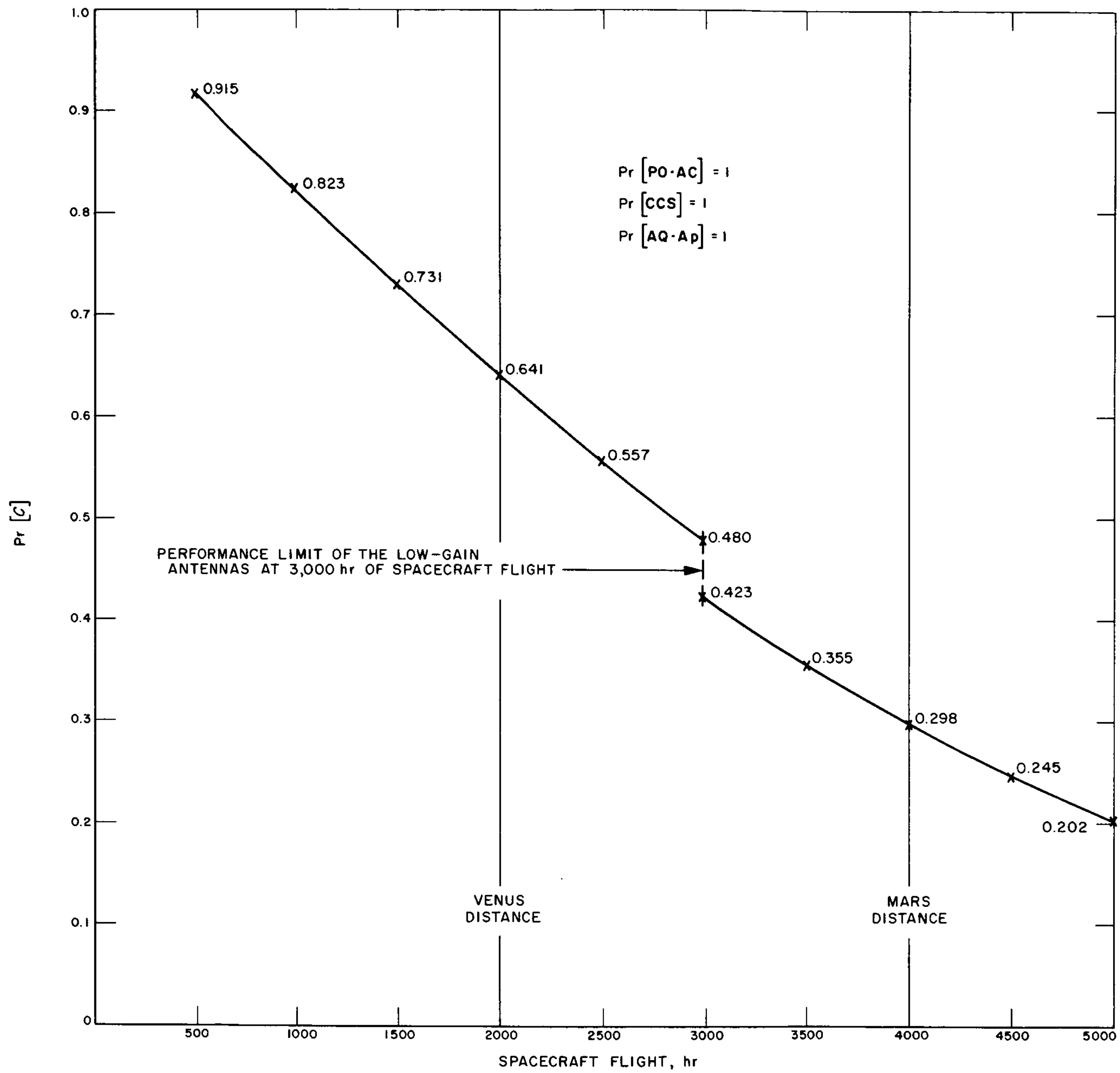


Fig. 35. Command function probability of success versus spacecraft flight hours

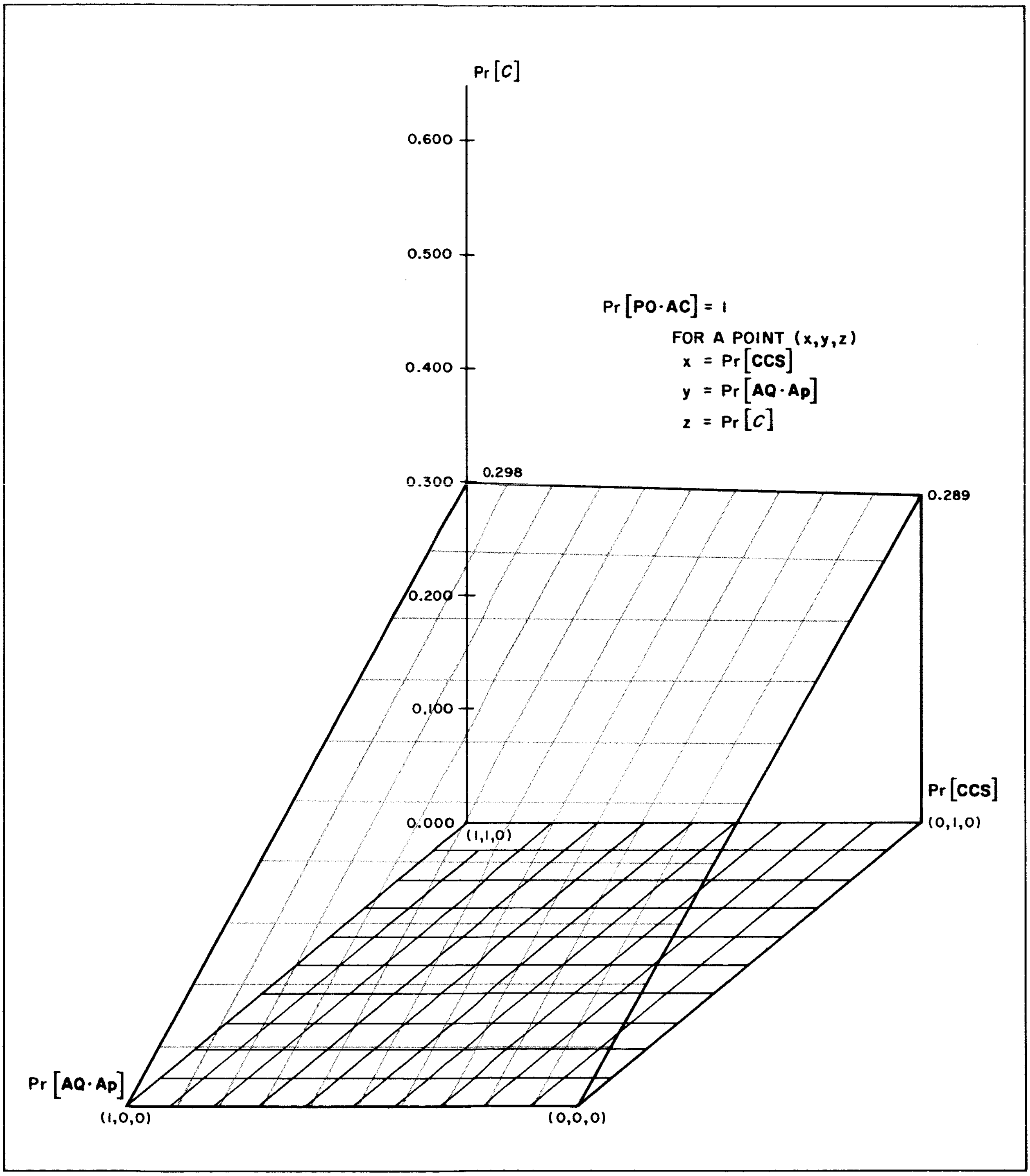


Fig. 36. Command function probability of success for Mars distance

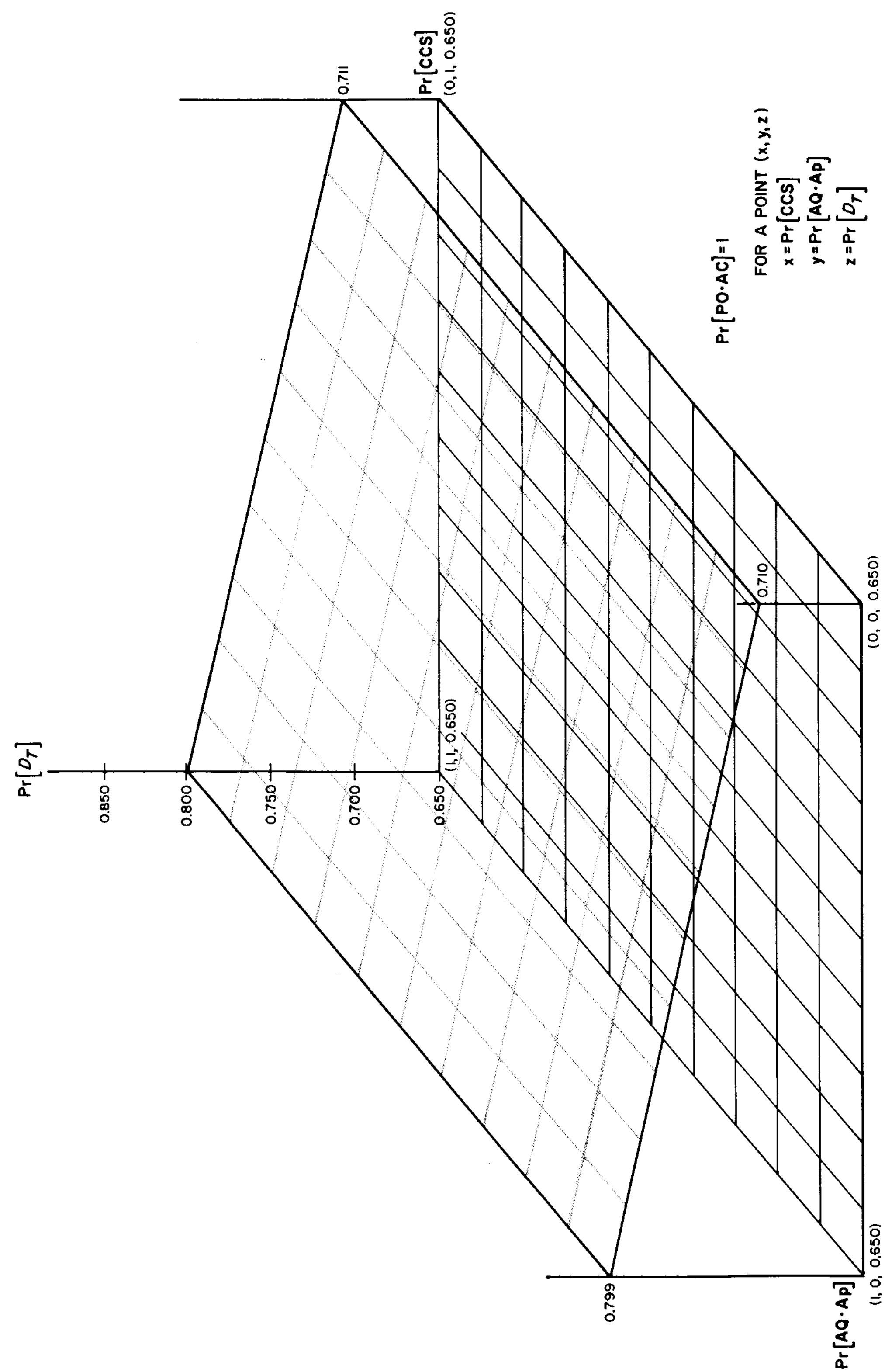


Fig. 37. Two-way doppler function probability of success for Venus distance

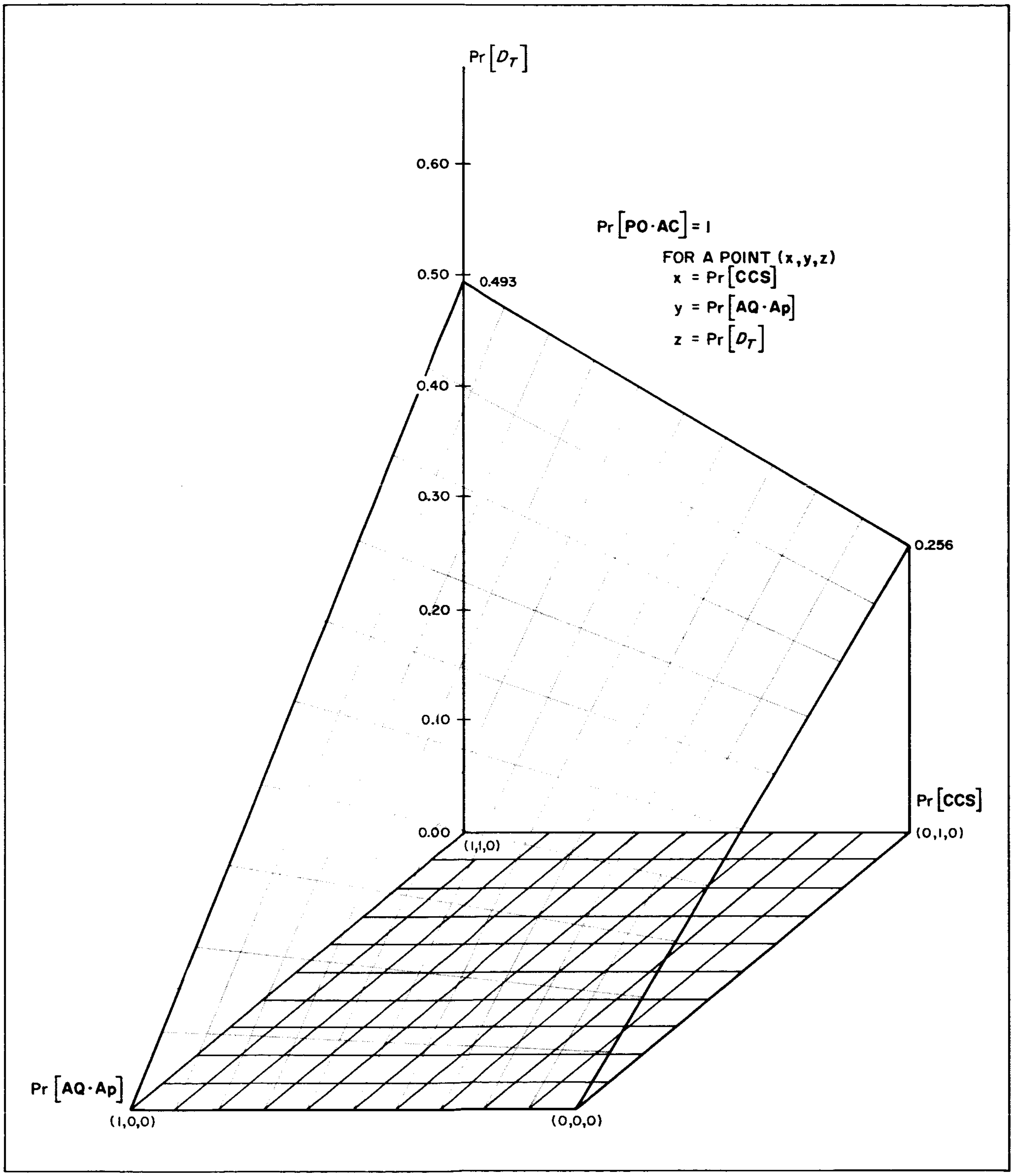


Fig. 38. Two-way doppler function probability of success for Mars distance

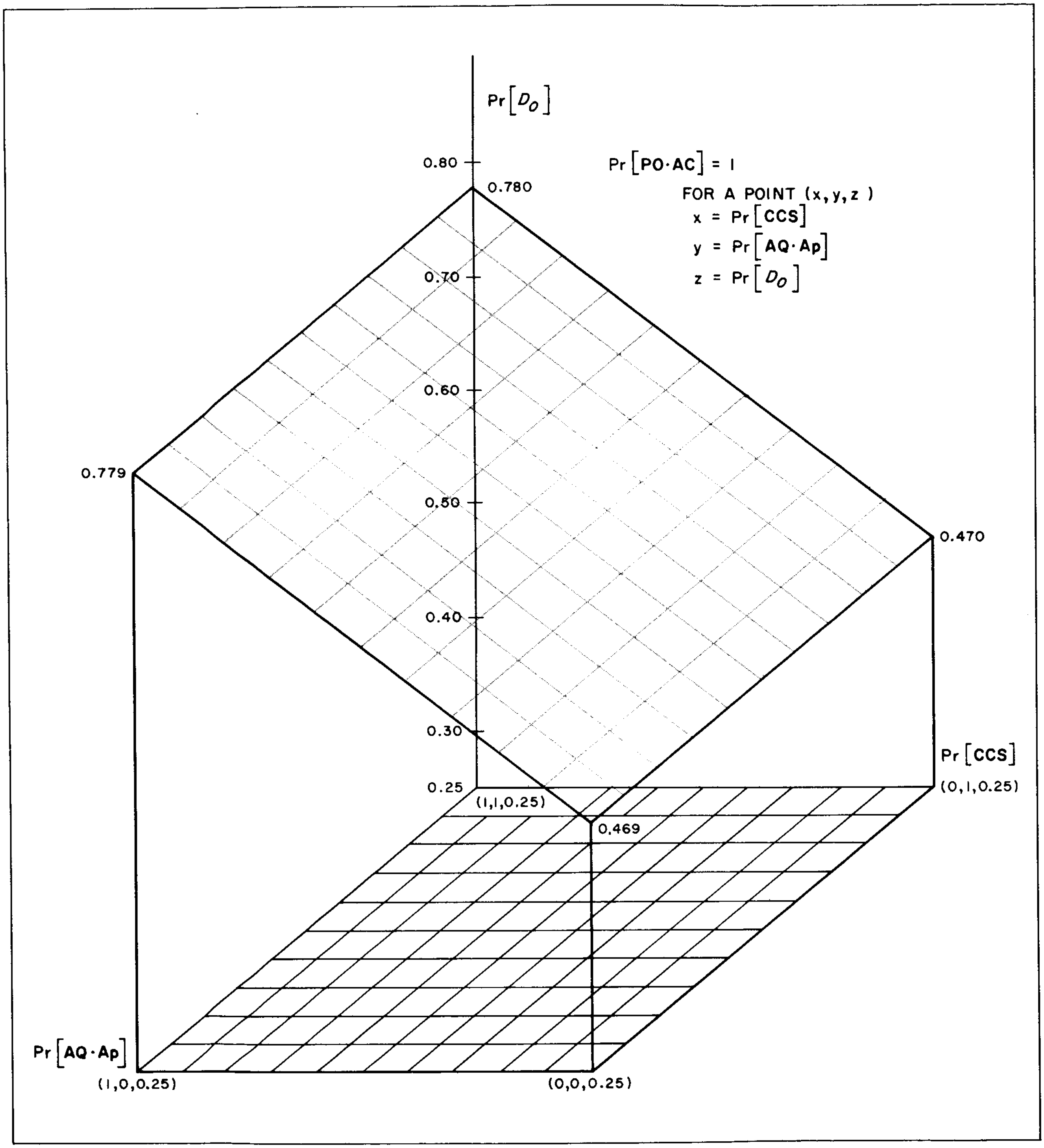


Fig. 39. One-way doppler function probability of success for Venus distance

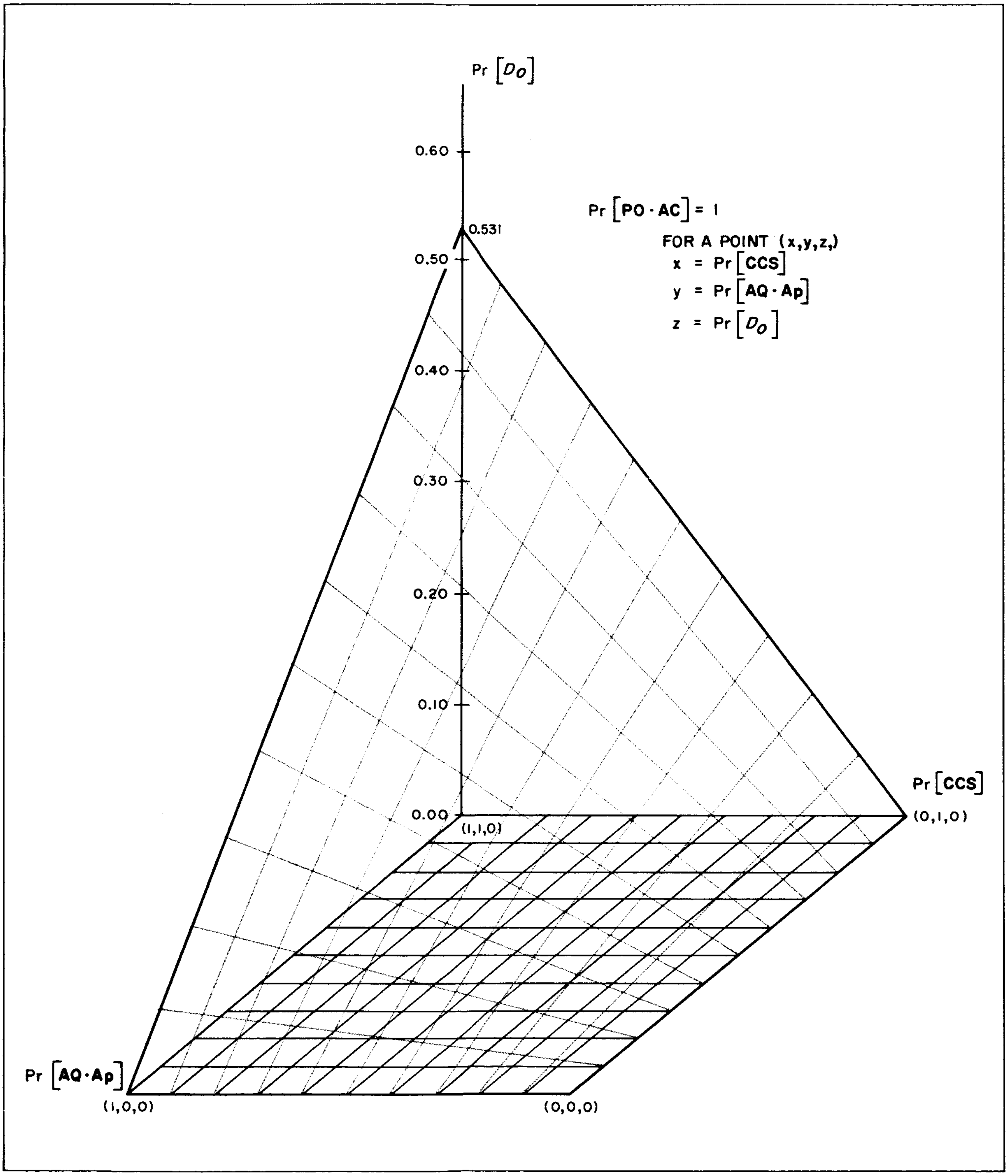


Fig. 40. One-way doppler function probability of success for Mars distance

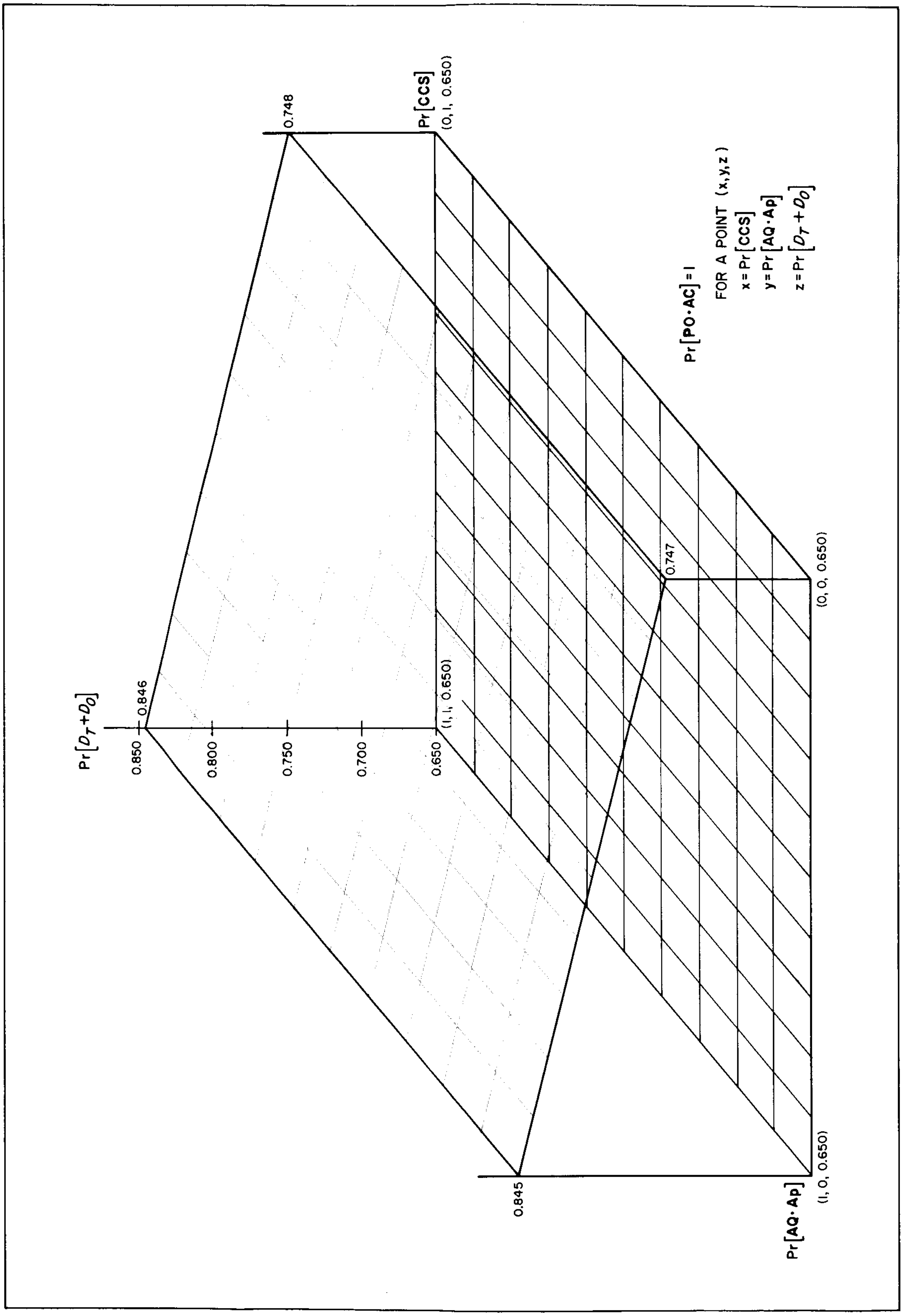


Fig. 41. One- or two-way doppler function probability of success for Venus distance

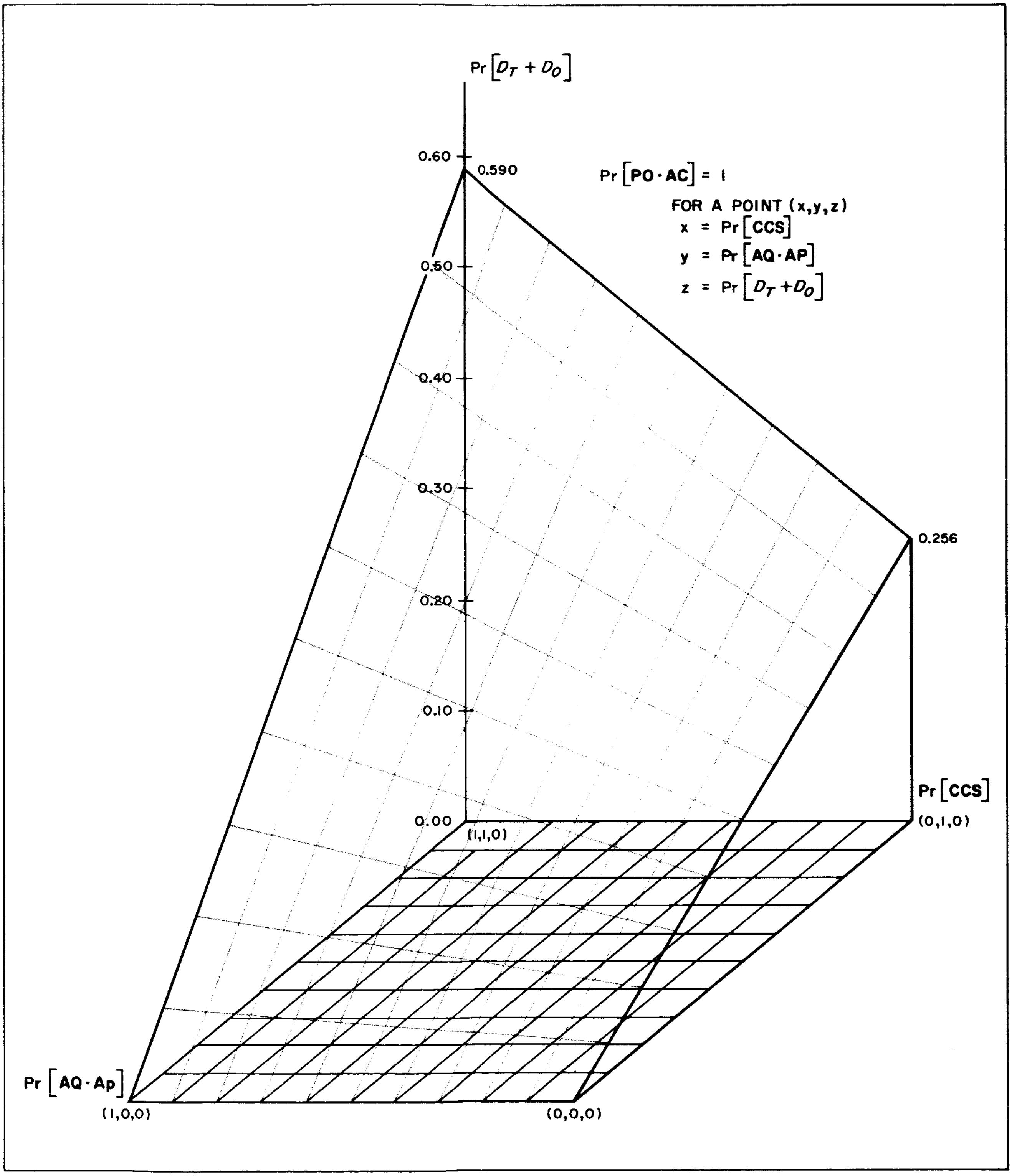


Fig. 42. One- or two-way doppler function probability of success for Mars distance

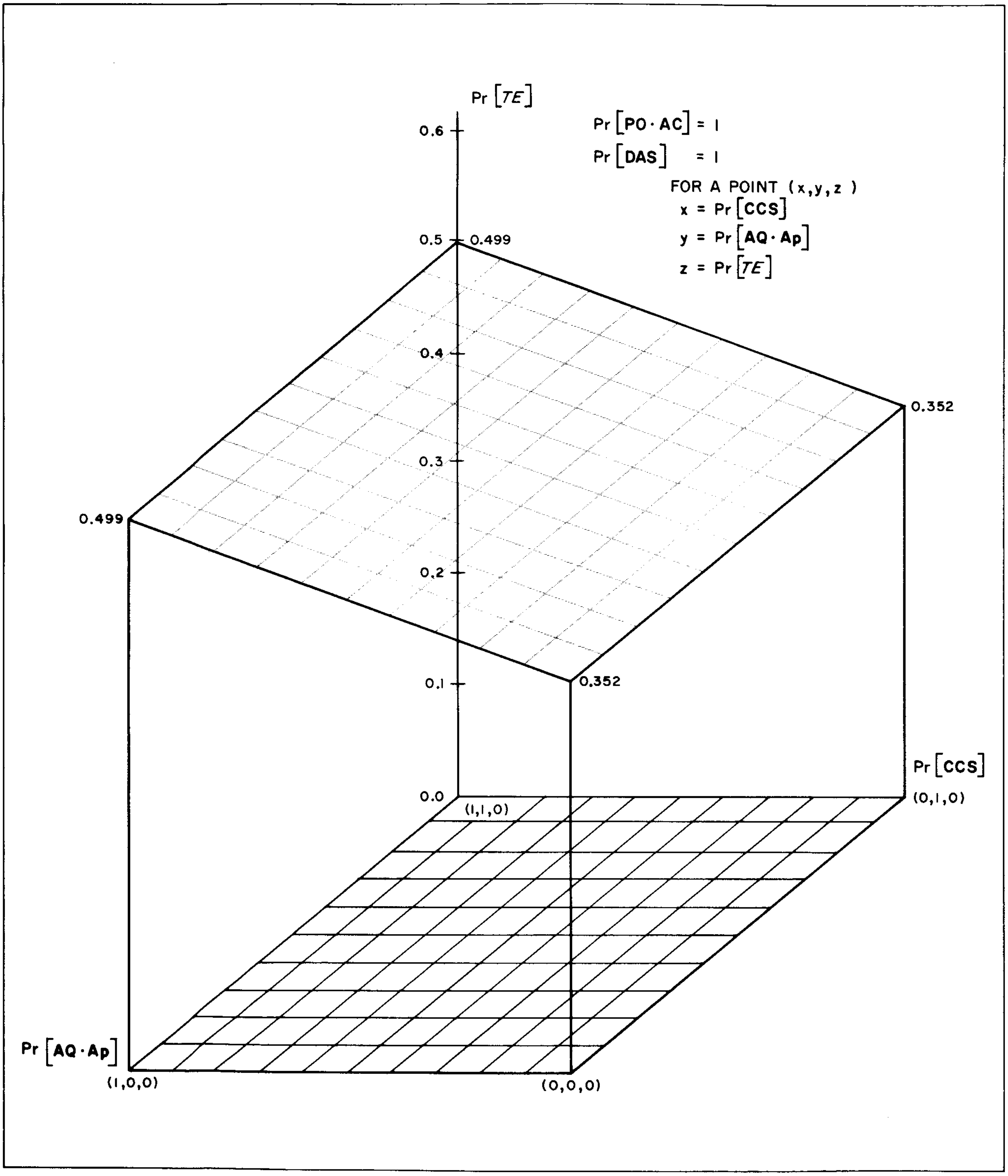


Fig. 43. Telemetry function probability of success for Venus distance

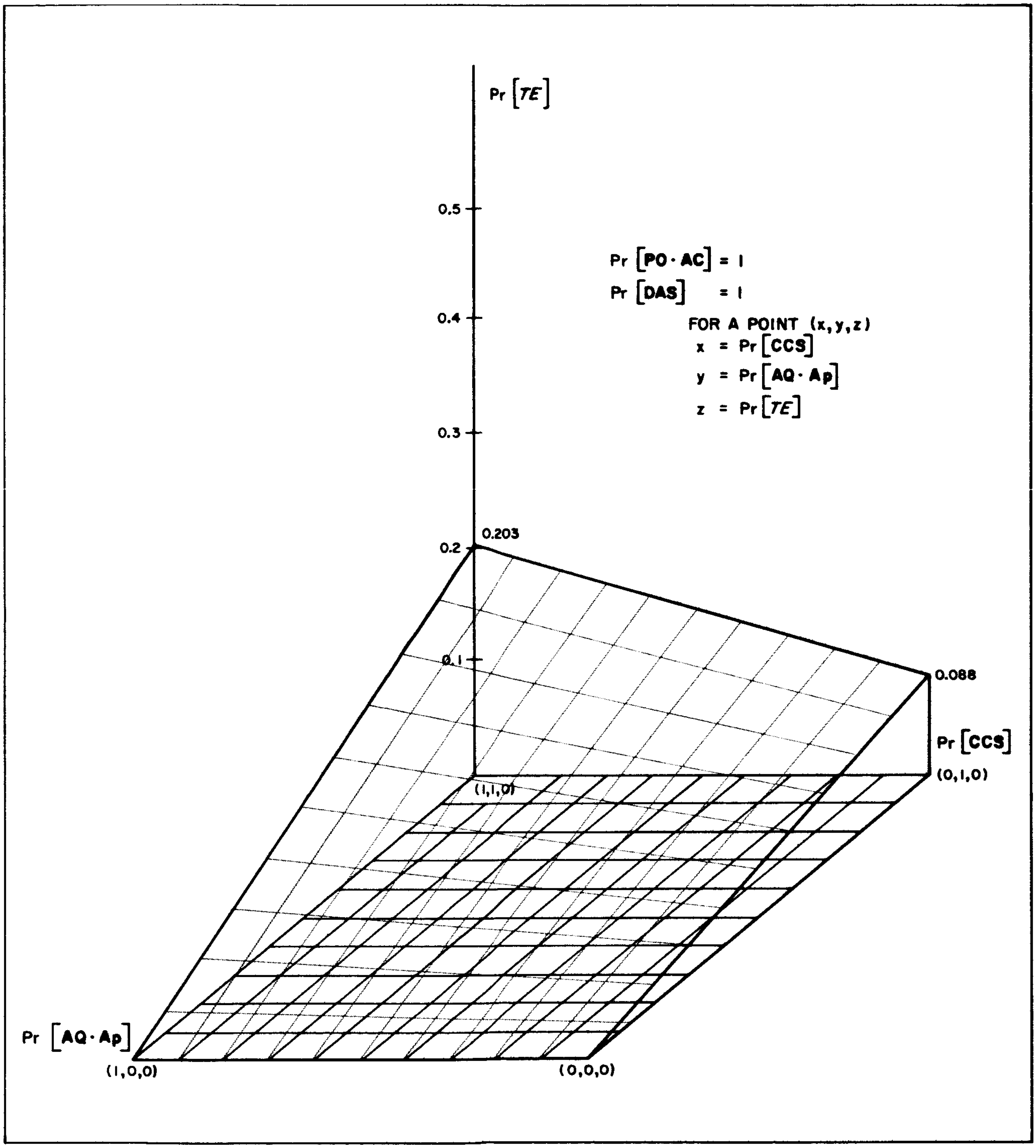


Fig. 44. Telemetry function probability of success for Mars distance

III. CONCLUSIONS, COMMENTS, AND RECOMMENDATIONS

A. Interpretation of Reliability Figures

Generally speaking, the reliability of a given system R_S is represented by a number between zero and one, once the mission of the system is defined. This number is used as an indication of the chance of success of the system mission and is widely accepted by reliability analysts and engineers. Surely this number is sufficient for reliability measurement or for the comparison of the relative reliabilities of two similar systems. However, it does not immediately lend itself to a quantitative interpretation by conventional standards. How much has really been gained from a system reliability improvement of 79% to 81%? Does a 0.1 reliability improvement bear the same meaning over the entire reliability spectrum? These questions cannot be answered merely by looking at the reliability figure.

Intuitively, a 0.1 increase in system reliability is rather easy to achieve in the lower reliability range. It becomes an impossibility when the initial reliability exceeds 0.9, or 90%, based on the definition of reliability.

A measure of the probabilistic nature which is closely related to the effort applied for reliability improvement may be obtained by a simple transformation of the reliability figure R_S expressed in

$$G = \frac{R_S}{1 - R_S} \tag{110}$$

where G is the ratio of the reliability to the nonreliability of the system. Simply, G represents the odds favoring the system's mission success. Hence, a reliability of 79% corresponds to favorable odds of 3.76, while an 81% reliability corresponds to odds of 4.26. An improvement in reliability from 79% to 81% may be thought of as a 13.3% increase in odds favoring the success of the mission.

A plot of Eq. (110) with G versus R_S is given in Fig. 45. On the same graph, per cent increase in odds as a function of a 0.1 reliability increment is also given. The significance of these curves is their close relationship to engineering effort, which can be measured in terms of money and time.

Consider that a spacecraft system to Venus is applied to these curves. A spacecraft possessing zero odds in favor of its success means that absolutely no effort is required. As tremendous engineering effort and manpower are spent, a mock-up version of the spacecraft can be built which possesses some chance of success. Improvement in reliability becomes less burdensome until the high reliability range is reached. Improving a spacecraft system from 0.8 to 0.9 reliability almost requires two spacecraft to be launched at the same time.

Although the curves in Fig. 45 may not necessarily be fair representations of the effort-reliability relationship, they do offer a guide in system engineering for making a proper compromise between system reliability improvement and effort spent.

B. Reliability Improvement in General

System failure originates with the failures of minute components within the system. Component reliability as related to its inherent failure rate and operating time has been discussed in Section II-A. Ironically, failures of components may be reduced if the failure rates of the components or the operating times are reduced.

Continuous advancement in electronic parts manufacturing processes and techniques enhances the reliability of future components. This statement has been verified throughout the past ten years by many independent studies and statistics such as the eight-year compilation of the United Kingdom Atomic Energy Research Establishment (Ref. 1). Technological breakthrough in rocketry that reduces the flight time of planetary travel to fractions of the present requirement may eventually be realized. Either factor can affect significantly the odds in favor of the success of future deep-space systems. Unfortunately, these types of improvement are time consuming and may not be achieved entirely during the next decade. During the present analysis, these two factors are assumed to be fixed. With these limitations imposed on the problem, the apparent methods to improve system reliability are the simplification and the application of redundancy techniques.

The simplification technique may be applied to circuit, functional block, and system levels. On the circuit and functional block levels, the primary task is the reduction in number of transistors or other parts. From the system point of view, it is the elimination of less essential equipment while retaining the primary functions that are required of the system. There is no fixed formula to govern the simplification of electronic equipment. The exercise of basic circuit knowledge, an understanding of the equipment, and, most of all, good judgement are emphasized.

In applying redundancy, system reliability can be improved at the expense of increases in size, weight, and cost of the equipment. When these factors become important, techniques must be developed to maximize the reliability improvement for any change in equipment design. The following paragraphs contain some general rules for optimizing redundancy applications.

1. Redundant Linkage

Statement: System reliability cannot be improved through unreliable redundant linkage.

It is not so simple to incorporate two or more equivalent functional groups of equipment in a parallel redundant fashion, especially in automatic machines. Some type of linking element is required to fulfill this purpose. Usually the redundant linking element consists of a selecting device, a decision-executing device, and a decision-making device. The selecting device may be a group of electronic or mechanical switches for making proper signal contacts. The decision-executing device is a control element which, upon receipt of signals from the decision-making device, alters the switch positions. The decision-making device may be a failure-detecting circuit that alarms the decision-executing device to act when failure is detected. Complexity of the linking element varies according to the type of functional block.

The general form of the reliability equation with two equivalent blocks in parallel is given by

$$R_S = (R_A + R_B - R_A \cdot R_B) \cdot R_L \tag{111}$$

where the expression within the parentheses is the ideal resultant of the parallel configuration of the two blocks with reliabilities R_A and R_B. The symbol R_L is the reliability of the linking element. This equation assumes that the failure of the linking element causes the system operation to fail. If R_L is low as compared to R_A or R_B, redundant effort is not justified. It is imperative that highly reliable linking elements be used in order to obtain reliability improvement through redundancy.

2. Choice of Redundant Equipment

Statement: If redundancy can be made for only one out of many functional blocks, the proper choice is to select the lowest reliability unit, provided that all other factors are the same.

Consider a system of two independent functional blocks A and B with reliabilities R_A and R_B, respectively. Because of other constraints, redundancy may be applied to only one of the two units. The resultant system reliability may be expressed by Eq. (112) or (113), depending upon the unit chosen. Thus Eq. (112) represents the system reliability after redundancy has been applied to unit A, while Eq. (113) represents the system reliability after redundancy has been applied to unit B. The redundant linkage is considered perfect.

$$R_{SA} = (2\,R_A - R_A^2) \cdot R_B \tag{112}$$

$$R_{SB} = (2\,R_B - R_B^2) \cdot R_A \tag{113}$$

Assume that Eq. (112) is the right choice such that

$$R_{SA} - R_{SB} > 0 \tag{114}$$

Substituting Eq. (112) and (113) into Eq. (114) yields

$$2\,R_A \cdot R_B - R_A^2 \cdot R_B - 2\,R_A \cdot R_B + R_A \cdot R_B^2 > 0$$

or

$$R_A \cdot R_B \cdot (R_B - R_A) > 0 \tag{115}$$

Since R_A and R_B are both positive quantities, if follows that the Eq. (115) can be fulfilled only when R_A is less than R_B. If R_A is equal to R_B, either selection is acceptable.

3. Redundancy Applied to a Group of Blocks

Statement: Highest reliability can be achieved if redundancy is applied on the most elementary basis.

To clarify this statement, consider the two possible configurations of two units for redundant mechanization. Figure 46(a) may be called the over-all system redundancy (*method a*) and Fig. 46(b), the elementary group redundancy (*method b*).

Let R_A and R_B be the reliabilities of units A and B, respectively. The system reliability for *method a* becomes

$$R_{SA} = 2\,R_A \cdot R_B - (R_A \cdot R_B)^2 \tag{116}$$

On the other hand, the system reliability for *method b* is

$$R_{SB} = (2\,R_A - R_A^2) \cdot (2\,R_B - R_B^2) = 4\,R_A \cdot R_B - 2\,R_A^2 \cdot R_B - 2\,R_A \cdot R_B^2 + (R_A \cdot R_B)^2 \tag{117}$$

If a difference between R_{SA} and R_{SB} is taken as indicated in Eq. (118), then

$$\begin{aligned} R_{SB} - R_{SA} &= 2\,R_A \cdot R_B - 2\,R_A^2 \cdot R_B - 2\,R_A \cdot R_B^2 + 2(R_A \cdot R_B)^2 \\ &= 2\,R_A \cdot R_B\,[1 - (R_A + R_B - R_A \cdot R_B)] \end{aligned} \tag{118}$$

It is noted that the term $(R_A + R_B - R_A \cdot R_B)$ cannot be greater than unity since both R_A and R_B are positive and are not greater than unity. This leads to the conclusion that $R_{SB} \geq R_{SA}$ and that *method b* is the preferred method. It can be shown that the elementary redundancy method is superior even though more than two functional units are involved.

There are occasions when *method a* is used instead of the preferred method. Observing Fig. 46 discloses that one additional linkage is required for *method b*. The reliability of this linkage must also be considered.

Redundancy can also be applied to the components level in addition to the functional block level. Two components may be connected in series or in parallel to improve the failure rate of the components. The selection of the redundant type is entirely dependent upon the type of failure mode most likely to occur. Consider a diode that has two failure modes, namely, fail open and fail short, and let λ_O and λ_S be the failure rates for the respective modes. The portions of the reliability of the diode applying to open failure and shorting failure are illustrated by Eq. (119) and (120):

$$R_O = e^{-\lambda_O T} \tag{119}$$

$$R_S = e^{-\lambda_S T} \tag{120}$$

Then the reliability of the diode is equal to

$$R_D = e^{-(\lambda_O + \lambda_S) T} = R_O \cdot R_S \tag{121}$$

Consider the two possible redundant connections given in Fig. 47. The reliability for the series connection is given by

$$R_A = R_O^2 \cdot (2\,R_S - R_S^2) = R_O \cdot R_S \cdot [(2 - R_S) \cdot R_O] \tag{122}$$

On the other hand, the reliability for the parallel configuration is

$$R_B = R_S^2 \cdot (2\,R_O - R_O^2) = R_O \cdot R_S \cdot [(2 - R_O) \cdot R_S] \tag{123}$$

Observing these two equations leads to the following conclusions:

1. If

$$(2 - R_S) > \frac{1}{R_O} \tag{124}$$

series redundancy is of advantage.

2. If

$$(2 - R_O) > \frac{1}{R_S} \tag{125}$$

parallel redundancy is of advantage.

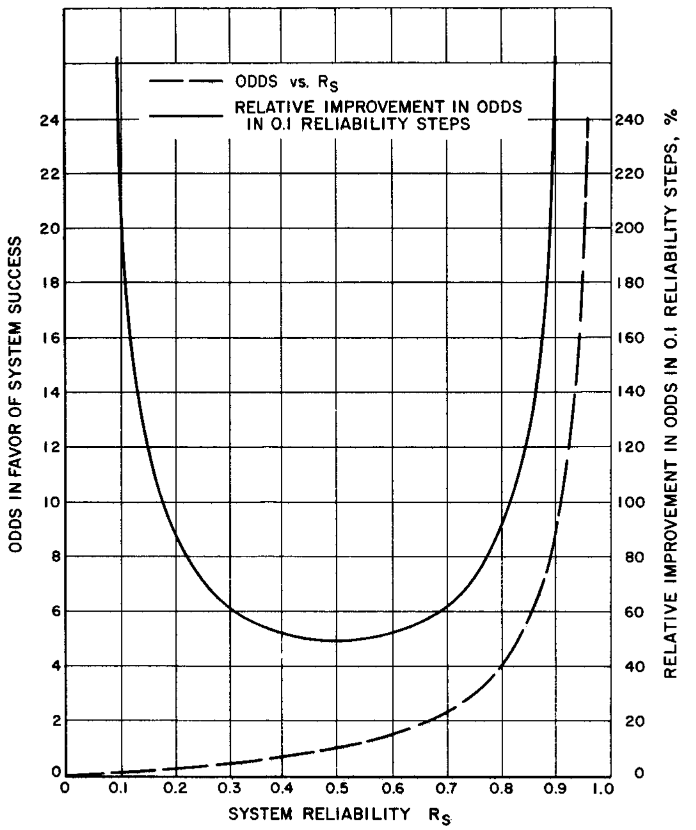


Fig. 45. Odds favoring the system mission success versus R_S

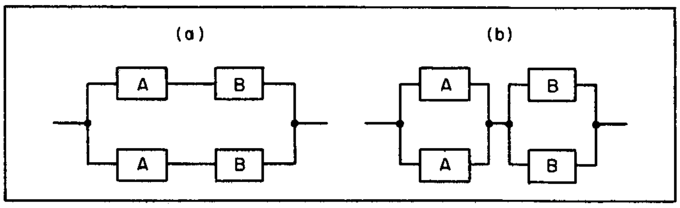


Fig. 46. Two methods of redundant mechanization
a. Redundancy in group arrangement
b. Redundancy in minute element arrangement

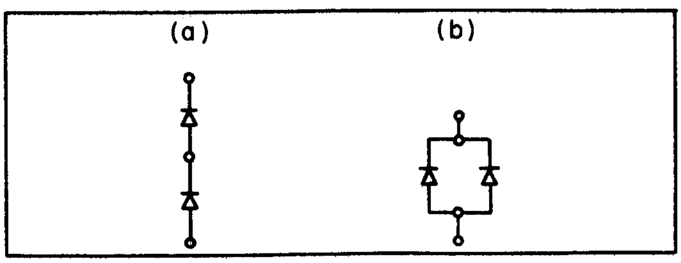


Fig. 47. Two methods of diode redundant connections
a. Series connection
b. Parallel connection

3. If neither Eq. (124) nor (125) is satisfied, none of the component redundant methods should be applied.

It can be proved that Eq. (124) and (125) can be reduced to Eq. (126) and (127), respectively, because of the fact that $(\lambda \cdot T)$'s are small for a single component.

$$\lambda_S > \lambda_O \tag{126}$$

$$\lambda_S < \lambda_O \tag{127}$$

Another component redundant method is the quad configuration that is illustrated in Fig. 48.

These methods warrant reliability improvement regardless of their failure modes.

C. Reliability Comparison Between *Mariner A* and *Mariner B* Communication Systems

This analysis cannot be considered complete unless some form of reliability comparison between the *Mariner A* and the *Mariner B* communication systems is performed. Functional reliabilities of the *Mariner A* spacecraft communication system have been obtained by Bourke in his Report, *A Reliability Analysis Method for Complex Systems*, (Ref. 2). In the *Mariner B* analysis, every attempt has been made to conform to the *Mariner A* study in such a way that meaningful comparisons between the studies can be realized.

Comparisons will be made on a functional basis, based on the Venus mission. Explanations of various outcomes will also be attempted.

1. Command Function

Table 4 discloses some interesting figures on the command function for *Mariner A* and *Mariner B*.

Table 4. Command function comparison

Comparison parameters	Reliability	
	Mariner A	*Mariner B*
Command function, Pr [**AQ** · **Ap**] = 1	0.658	0.641
Command function, Pr [**AQ** · **Ap**] = 0	0.499	0.640
Command subsystem: Command detector, decoder, and matrix	0.779	0.758
Radio equipment, Pr [**AQ** · **Ap**] = 1	0.844	0.846
Radio equipment, Pr [**AQ** · **Ap**] = 0	0.641	0.845
Command detector	--	0.733

The probability of success of the *Mariner A* command function was found to be 0.658 when the reliability of the Earth acquisition function is unity and was 0.499 with zero reliability for the same external dependent. This is in contrast to the finding of the *Mariner B* command function which is relatively independent of the acquisition function. The command function reliabilities between the two extremes are 0.641 and 0.640, correspondingly. The *Mariner B* command function independence of the acquisition function is understandable since an additional low-gain antenna is used on the spacecraft. The disability of the high-gain antenna, because of the failure of the acquisition function, is backed up by this low-gain antenna with sacrifice in performance. This independence is certainly a desirable feature and can be considered an improvement over the *Mariner A* spacecraft.

Under the conditions Pr [**AQ** · **Ap**] = 1 and Pr [**CCS**] = 1, the command function reliability of the *Mariner B* system is 0.641 as compared to 0.658 for the *Mariner A* system. As can be seen from Table 4, this decrease in reliability is due to the large contribution of the command subsystem. The reliability of the command block is actually lower, despite the fact that redundancy has been applied in the command detector unit. There are two major reasons for this deficiency. The first reason is the lower reliability (0.733) found in the *Mariner B* detector unit. Although the actual figure for this corresponding unit in the *Mariner A* is not available, it is at least 0.85, judging from the command subsystem reliability figure of 0.779 (see Table 4). The second reason is the increased capability of the *Mariner B* command decoder. It provides more command signals to other portions of the spacecraft by increasing the complexity of the decoding circuitry.

2. Two-Way Doppler Function

To compare this function for *Mariner A* and *Mariner B*, it is advantageous to construct three-dimensional plots for both systems. These plots are given in Fig. 49. From these drawings, the improvement of the *Mariner B* system over that of the *Mariner A* is apparent. The reliability of the two-way doppler function is practically not affected by the acquisition function for the reasons given in Section III-A. The CC&S function variation also contributes only a slight change in the function reliability for the *Mariner B* as compared to a 35% change in the *Mariner A* function. This improvement is derived from incorporating partial successes of various switches into the analysis method.

Under the optimum condition, the two-way doppler function of the *Mariner B* has a reliability of 0.800 against the 0.727 for the *Mariner A* spacecraft, a 50% increase in odds. This improvement is credited to the superior mechanization of the radio subsystem, the better redundant arrangements, and an additional antenna.

3. Telemetry Function

The telemetry function may be considered as the joint function of the radio subsystem and the telemetry subsystem. Since the radio portions have already been compared, it is only necessary to compare the reliabilities of the data encoders.

The *Mariner B* telemetry subsystem has a reliability of 0.59 as compared to 0.647 calculated for the *Mariner A*, measuring under optimum conditions. The reliability of the *Mariner B* data encoder, according to he above indicated figures, is actually lower. Although the redundancy configurations used in this encoder do not show any appreciable improvement, they certainly do not lower its reliability. The reason for this unhappy finding seems to be the difference in data rate and mode switch reliabilities. The total reliability of these switches obtained in the *Mariner A* system was 0.986 while the reliability of the equivalent *Mariner B* switching group was 0.865. This is undoubtedly a big difference. If the same 0.986 figure were used in the *Mariner B* calculation, the data encoder reliability would have been 0.673, a number more favorable for the recent system.

It is believed that the *Mariner B* data encoder has higher flexibility as far as the operation and data modes are concerned. The improvement in flexibility is achieved at the expense of lowering function reliability.

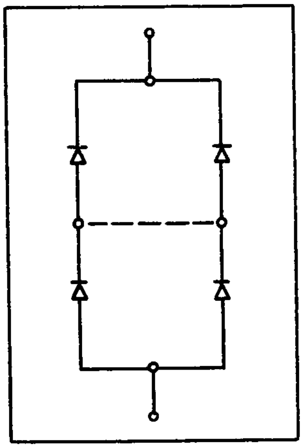


Fig. 48. Diode quad configuration

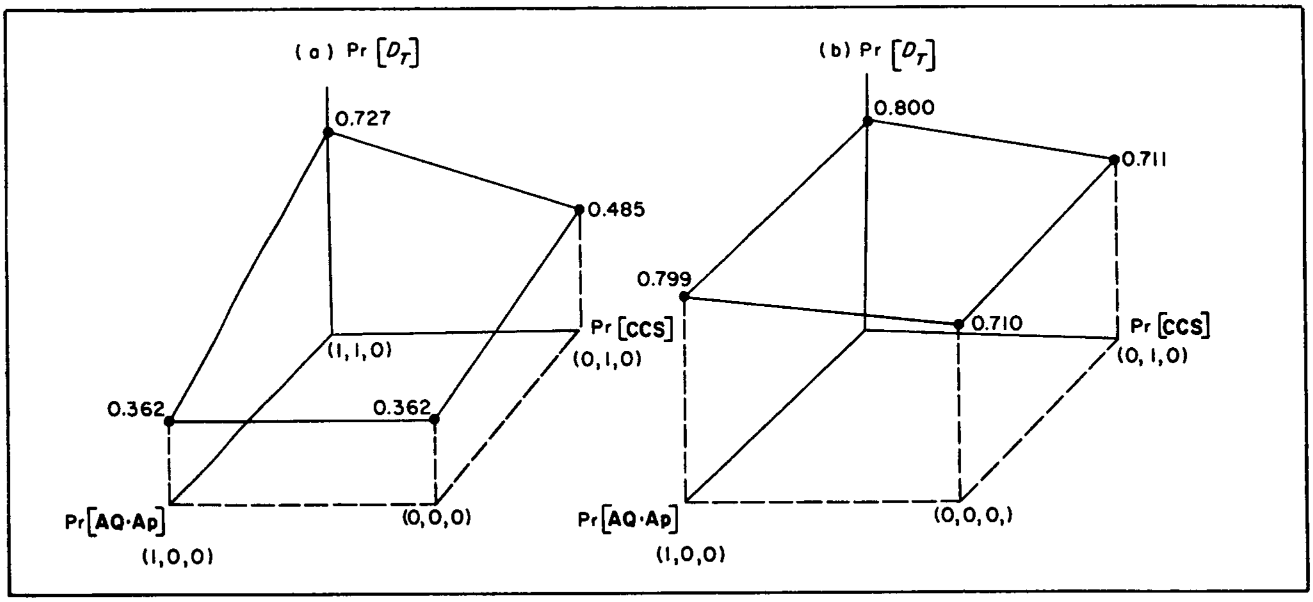


Fig. 49. Two-way doppler function probability of success

a. *Mariner A*
b. *Mariner B*

Although different analytical methods were used to obtain the reliabilities of the multiplexing equipment for *Mariner A* and *Mariner B*, the results obtained were of the same order of magnitude. The previous comparisons are typical examples. Other functions, if compared, will exhibit similar results.

No comparison is made by using the outcomes obtained for the Mars mission. As a result of the disability of the *Mariner B* low-gain antennas, a comparison made on this basis is not appropriate.

D. Comments and Recommendations

1. Command Subsystem

The command subsystem plays a rather important role in the success of the telecommunication mission of the *Mariner B* spacecraft, both directly and indirectly. A highly reliable command unit can make the telecommunication functions relatively unaffected by the external dependent, the Central Computer and Sequencer system, and it can allow utilization of the full potential of redundant mechanization in other portions of the communication system.

The command subsystem under consideration is well constructed. Redundancy is practiced following the rules of good sense. The weakest link in the command subsystem is the decoding unit. It consists of a vast number of diode-transistor logic elements which are used to discriminate and to select the incoming command code words. As can be seen, largely because of its multi-inputs and outputs construction, gross application of redundancy to this unit is not practical. To improve its reliability figure, circuit simplification seems to be the remaining method available. It has been noticed that many diode-transistor logic elements with a fan-in of two or three are used with this unit. As a matter of fact, they are used quite frequently throughout the entire subsystem. Since logic elements of this type may be designed by using resistors and transistors, they may replace all the diodes, which have a much higher failure rate than the composition resistors. However, such redesign is not recommended for the decoding elements with a fan-in of eight. The imperfections of the components rule out such a possibility.

Another factor contributing to the increase of the subsystem's failure rate is the use of tantalytic capacitors in analog circuits. In many instances, these capacitors are used merely for filtering and decoupling purposes where tolerances are in no way critical. Component redundant techniques can be applied here. To obtain the maximum advantage, a speedup testing of the tantalytic capacitor to determine its relative mode of failure is recommended. Depending on the test results, one of the three component redundant methods described previously may be adopted.

2. Radio Subsystem

The radio subsystem includes three main portions: the antenna portion, the receiver portion, and the transmitter portion. The use of redundant and backup equipment in all of these portions results in a superior design in the reliability measure. This design is very important since this subsystem is so critical that the success of the entire mission depends upon its success.

In the existing design, the switching command applied to the transmitting portion of this subsystem is obtained only from the command subsystem. In the event that one-way communication (from spacecraft to Earth) is operative, the transmitters and power amplifier become nonredundant because of the absence of a command signal. It is recommended that a CC&S command be introduced here for backup control purposes. With such a provision, an increase in reliability of 0.03 for the one-way doppler function can be realized, provided that Pr [**CCS**) is unity.

In the *Mariner B* transponder, no diplexors are being used. The routing of incoming and outgoing signals is directed by means of voltage-operated circulating RF switches. Consider the two switches K_3 and K_6. Because of the desired two-way communication paths, these switches are necessarily controlled separately. In the event that a switch might fail to operate properly, there is a chance that the output of the transmitter amplifier might be shorted to the input of a receiver. Would such an incident permanently damage the operation of this transponder? Or should this scheme be amended to avoid this possibility?

The use of three antennas in the spacecraft benefits more in performance than in reliability improvement. A computation was made on the command function (Venus distance) by assuming that the low-gain antenna A_1 does not exist. The probability of success for this situation is

$$\Pr[C] = 0.616 + 0.001 \Pr[\mathbf{CCS}] + 0.030 \Pr[\mathbf{AQ} \cdot \mathbf{Ap}] + 0.001 [\mathbf{CCS}] \cdot \Pr[\mathbf{AQ} \cdot \mathbf{Ap}] \tag{128}$$

This equation indicates that the low-gain antenna A_1 makes a significant contribution only when the Acquisition and the Antenna positioning events are less likely to succeed.

3. Telemetry Subsystem

It is disappointing to note the low reliability figures in the telemetry subsystem. The encoder has a 0.590 reliability for the Venus distance and a 0.344 reliability for the Mars distance. These figures include measures of partial successes in the commutator element. These figures are hardly encouraging with equipment of such importance.

The weakest link seems to be the power-supply unit for this subsystem. Factors that contributed to the low reliability figure were the power transistors and capacitors in the voltage-regulating circuits. It is realized that all the analog circuits which require tight power-supply tolerances are energized only by the ±20v lines, while all of the digital circuits are powered by the ±6v supplies. In digital circuitry, voltage-supply variations are less critical and a reasonable tolerance can be designed into the circuits. Consequently, the power-supply regulation problems are relatively unimportant. It is suggested that the regulating circuits in the ±6v power supplies be simplified. The reduction in size and weight of the subsystem as a result of this simplification can be of further benefit by applying redundancy to all the filter capacitors and power transistors.

Other areas with comparatively low reliabilities such as the rate-selecting mechanism and operational mode logic blocks utilize a sizable number of electro-mechanical relays. Although the switching cycles of these relays are quite infrequent, the inherent low reliability resulting from mechanical motion in relay cannot be overstressed.

Investigation of solid-state devices to replace mechanical relays is proposed. If successful, not only can the reliability of these portions be improved, but also possible arcing at relay contacts under the space environment can be eliminated.

As pointed out in Section III-B, redundancy can improve equipment reliability only when the linking circuit has a high reliability figure compared to the equipment to be combined in parallel. In this subsystem, the rate generator has a reliability figure of 0.979, while the redundant linkage has a reliability of 0.976. There is no improvement in reliability over the non-redundant arrangement by introducing the redundant circuitry.

The present mechanization of the PN generator and the analog-to-digital converter redundancies is applied by group arrangement rather than by minute element arrangement. It is advisable to connect the two analog-to-digital converters in parallel, independent of the other groups. In fact, there is no apparent improvement by redundance in the PN generator and decoder since their reliability figures are quite impressive.

NOMENCLATURE

General Notation

$\mathcal{A}$	an event used in Bayes' theorem
$\mathcal{B}_1$, $\mathcal{B}_2$	success events for the input functions of a switching configuration
$\mathcal{B}_3$, $\mathcal{B}_4$	success events for the output functions of a switching configuration
B_1	defined by Eq. (32)
B_2	defined by Eq. (33)
B_{21}	defined by Eq. (50)
B_3	defined by Eq. (34)
B_4	defined by Eq. (35)
B_5	defined by Eq. (51)
B_6	defined by Eq. (52)
B_{61}	defined by Eq. (53)
B_{62}	defined by Eq. (54)
$\mathcal{C}$	success event of the control function
$\overline{\mathcal{C}}$	failure event of the control function
C_1	defined by Eq. (46)
C_2	defined by Eq. (47)
$\mathcal{E}_j$	an event within a sample space
$\overline{\mathcal{E}}_j$	an event complementary to event $\mathcal{E}_j$
F	success event of the feedback mechanism
f	failure = t/τ
G	odds favoring the system mission success
H	defined by Eq. (C-3)

NOMENCLATURE (Cont'd)

h_i	constants or coefficients describing the probability of success for the one-way doppler function
$\mathcal{I}$	the universal set under discourse
J_0	defined by Eq. (D-13)
$\mathcal{K}_i$	success event of switch i
L_1	defined by Eq. (57)
L_2	defined by Eq. (58)
$\mathfrak{M}$	success event of the bistable memory element
$\mathfrak{M}_{ij}$	partial success for M, where i = 1 or 2 and j = 1 or 2
$\mathfrak{N}$	the null set
N	number of good components of the same kind
N_0	initial number of good components
n	total number of essential parts within a block
P_n	probability of exactly n channels operating successfully
Q_1	defined by Eq. (60)
Q_2	defined by Eq. (61)
R_A	reliability of block A
R_{amp}	reliability of the amplifier
R_B	reliability of block B
R_{BS}	reliability of the bucking supply
R_{Ci}	reliability of channel i
R_D	reliability of the diode
R_I	reliability of the current source
R_L	reliability of the linking element
R_O	portion of the reliability of a component applying to open-circuit failure

NOMENCLATURE (Cont'd)

R_S	portion of the reliability of a component applying to short-circuit failure
R_{sensor}	reliability of the sensor
R_{SQ}	reliability of the sequencer
R_{SW}	reliability of the switch
R_{SX}	the resultant reliability of a series of independent blocks when block X is made redundant (X = block A or block B)
$\mathcal{S}$	success event for a system function
$\overline{\mathcal{S}}$	failure event for a system function
S	stress factor
T	defined by Eq. (B-2)
T'	defined by Eq. (B-9)
t	total system operating time
U	defined by Eq. (B-3)
U · V	defined by Eq. (B-5)
U'	defined by Eq. (D-9)
U″	defined by Eq. (F-7)
U″ · V″	defined by Eq. (F-9)
V	defined by Eq. (B-4)
V'	defined by Eq. (D-10)
V″	defined by Eq. (F-8)
W	defined by Eq. (A-2)
W'	defined by Eq. (D-8)
W' · U'	defined by Eq. (D-16)
W' · V'	defined by Eq. (D-17)
W' · U' · V'	defined by Eq. (D-18)

NOMENCLATURE (Cont'd)

y_i, y_i'	constants or coefficients describing the probability of success of the command function
z_i	constants or coefficients describing the probability of success for the two-way doppler function
α_1	defined by Eq. (D-19)
α_2	defined by Eq. (D-20)
β_1	defined by Eq. (D-21)
β_2	defined by Eq. (D-22)
γ_1	defined by Eq. (D-23)
γ_2	defined by Eq. (D-24)
λ	constant failure rate of components of the same kind
λ_O	constant failure rate applying to open-circuit failure
λ_S	constant failure rate applying to short-circuit failure
μ_0	defined by Eq. (E-12)
μ_1	defined by Eq. (E-13)
μ_2	defined by Eq. (E-14)
τ	mean time between failures

Spacecraft Telecommunication Events

A_1	success event of low-gain antenna 1
A_2	success event of low-gain antenna 2 (preferred)
A_{HG}	success event of high-gain antenna
C	success event of the command function
C_D	success event of command detection and decoding function

NOMENCLATURE (Cont'd)

C'_D	success event of command detection and decoding function including the reliability of various control switches
D_O	success event of the one-way doppler function
D_T	success event of the two-way doppler function
G_1	success event of the command input amplifier
G_2	success event of command detector 1
G_3	success event of command detector 2
G_4	success event of the power selector
G_5	success event of the decoder access switch
G_6	success event of the command decoder
G_7	success event of transmitter 1 including transfer switch
G_8	success event of transmitter 2 including transfer switch
G_9	success event of amplitron 1 as an amplifier when energized
G'_9	success event of amplitron 1 as a transmission line when de-energized
G_{10}	success event of amplitron 2 as an amplifier when energized
G'_{10}	success event of amplitron 2 as a transmission line when de-energized
G_{11}	success event of auxiliary oscillator associated with transmitter 1
G_{12}	success event of auxiliary oscillator associated with transmitter 2
K_i	success event of switch K_i
K_{i1}	event of RF switch K_i making contact with one direction when the control to this switch has failed
K_{i2}	event of RF switch K_i making contact with the other direction when the control to this switch has failed
M	success event of memory element controlling switching pair K_1 and K_2
M_{11}	event in which switches K_1 and K_2 are caused to make contact with one position when M has failed

NOMENCLATURE (Cont'd)

M_{12}	event in which switches K_1 and K_2 are caused to make contact with the other position when M has failed
M_{21}	event in which switches K_1 and K_2 are locked to one position when the control function to the memory element M has failed
M_{22}	event in which switches K_1 and K_2 are locked to the other position when the control function to the memory element M has failed
N_1	success event of rate generator 1
N_2	success event of rate generator 2
N_3	success event of rate generator power switch
N_{31}	event in which rate generator 1 is energized when N_3 has failed
N_{32}	event in which rate generator 2 is energized when N_3 has failed
N_4	success event of the *OR* gates for the rate generators' output
N_5	success event of rate-selecting relays
N_6	success event of selecting switches for PN generators
N_{61}	event in which PN generator 1 is energized when N_6 has failed
N_{62}	event in which PN generator 2 is energized when N_6 has failed
N_7	success event of PN generator 1
N_8	success event of PN generator 2
N_9	success event of PN decoder 1
N_{10}	success event of PN decoder 2
N_{11}	success event of modulator 1
N_{12}	success event of modulator 2
N_{13}	success event of analog-to-digital converter 1
N_{14}	success event of analog-to-digital converter 2
N_{15}	success event of output amplifier

NOMENCLATURE (Cont'd)

N_{16}	success event of multiplexing equipment
N_{17}	success event of data mode logic
N_{18}	success event of operational mode logic
N_{19}	success event of the OR gate for the analog-to-digital converters' output
N_{20}	success event of data flow gates
P_1	success event of transformer-rectifier unit for command detector 1
P_2	success event of transformer-rectifier unit for command detector 2
P_3	success event of either P_1 or P_2 and the power selector G_4
P_4	success event of transformer-rectifier unit for the radio subsystem
P_7	success event of amplitron power supply
P_8	success event of encoder power supply
R_1	success event of receiver 1
R_2	success event of receiver 2
R_t	success event of range-tracking function
S_1	success event of command detector selector
S_{11}	event of switch making contact with command detector 1 when the selector has failed
S_{12}	event of switch making contact with command detector 2 when the selector has failed
S_2	success event of receiver output selector
S_{21}	event of switch making contact with receiver 1 when selector S_2 has failed
S_{22}	event of switch making contact with receiver 2 when selector S_2 has failed
S_3	success event of transmitter power-selecting switch

NOMENCLATURE (Cont'd)

$(S_3 \cdot K_4)_1$ event that both S_3 and K_4 are in the proper positions for transmitter 1 making connection with the amplitron input when control has failed

$(S_3 \cdot K_4)_2$ event that both S_3 and K_4 are in the proper positions for transmitter 2 making connection with the amplitron input when control has failed

S_4 success event of amplitron power-selecting switch

S_{41} event that S_4 makes contact with amplitron 1 when control has failed

S_{42} event that S_4 makes contact with amplitron 2 when control has failed

S_W success event of the signal acquisition for the command function

S_X success event of signal acquisition and transmission functions

SK success event of control switching circuitry

T success event of transmission preparation function

TE success event of telemetry function

T_e success event of telemetry-encoding function

Spacecraft External Dependents

AC success event of attitude control

Ap success event of antenna positioning

AQ success event of Canopus acquisition

CCS success event of the Central Computer and Sequencer

DAS success event of the data automation system

E success event of the external dependents

PO success event of main power

RG success event of ranging subsystem

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APPENDIX A. Derivation of Pr $[S_{\overline{W}}]$ for the Command Function

The success event of the signal acquisition for the command function is

$$S_{\overline{W}} = P_4 \Big[\{R_2 \cdot (S_2 + S_{22}) \cdot [(\mathbf{CCS} + SK) \cdot (M + M_{11}) + \overline{\mathbf{CCS}} \cdot \overline{SK} \cdot M_{21}] + R_1 \cdot (S_2 + S_{21})$$

$$\cdot \; [(\mathbf{CCS} + SK) \cdot (M + M_{12}) + \overline{\mathbf{CCS}} \cdot \overline{SK} \cdot M_{22}] \cdot K_2\} \cdot K_1 \cdot A_2 \cdot K_7 + \{R_1 \cdot (S_2 + S_{21})$$

$$\cdot \; [(\mathbf{CCS} + SK) \cdot (M + M_{11}) + \overline{\mathbf{CCS}} \cdot \overline{SK} \cdot M_{21}] + R_2 \cdot (S_2 + S_{22}) \cdot [(\mathbf{CCS} + SK) \cdot (M + M_{12})$$

$$+ \; \overline{\mathbf{CCS}} \cdot \overline{SK} \cdot M_{22}] \cdot K_1\} \cdot K_2 \cdot \{(\mathbf{AQ}) \cdot \mathbf{Ap} \cdot A_{HG} \cdot (\mathbf{CCS} + SK) \cdot K_3 \cdot K_6 + A_1$$

$$\cdot \; [(\mathbf{CCS} + SK) + \overline{\mathbf{CCS}} \cdot \overline{SK} \cdot K_{32}] \cdot K_3\} \Big] \tag{A-1}$$

By letting

$$[\mathrm{W}] = \{(\mathbf{AQ}) \cdot \mathbf{Ap} \cdot A_{HG} \cdot (\mathbf{CCS} + SK) \cdot K_3 \cdot K_6 + A_1 \cdot [(\mathbf{CCS} + SK) + \overline{\mathbf{CCS}} \cdot \overline{SK} \cdot K_{32}] \cdot K_3\} \tag{A-2}$$

and by rearranging Eq. (A-1),

$$S_{\overline{W}} = P_4 \cdot \Big[\{ [A_2 \cdot K_1 \cdot K_7 \cdot R_2 \cdot (S_2 + S_{22}) + [\mathrm{W}] \cdot K_2 \cdot R_1 \cdot (S_2 + S_{21})] \cdot [(\mathbf{CCS} + SK) \cdot (M + M_{11})$$

$$+ \overline{\mathbf{CCS}} \cdot \overline{SK} \cdot M_{21}] \} + \{ [A_2 \cdot K_1 \cdot K_2 \cdot K_7 \cdot R_1 \cdot (S_2 + S_{21}) + [\mathrm{W}] \cdot K_1 \cdot K_2 \cdot R_2 \cdot (S_2 + S_{22})]$$

$$\cdot [(\mathbf{CCS} + SK) \cdot (M + M_{12}) + \overline{\mathbf{CCS}} \cdot \overline{SK} \cdot M_{22}] \} \Big] \tag{A-3}$$

The associated probability is

$$\Pr\,[S_{W}] = \Pr\,[P_4]\cdot\Big\{\Pr\{[A_2\cdot K_1\cdot K_7\cdot R_2\cdot(S_2+S_{22}) + [W]\cdot K_2\cdot R_1\cdot(S_2+S_{21})]\cdot[(\mathbf{CCS}+SK)$$

$$\cdot(M+M_{11}) + \overline{\mathbf{CCS}}\cdot\overline{SK}\cdot M_{21}]\} + \Pr\{[A_2\cdot K_1\cdot K_2\cdot K_7\cdot R_1\cdot(S_2+S_{21}) + [W]$$

$$\cdot K_1\cdot K_2\cdot R_2\cdot(S_2+S_{22})]\cdot[(\mathbf{CCS}+SK)\cdot(M+M_{12}) + \overline{\mathbf{CCS}}\cdot\overline{SK}\cdot M_{22}]\}$$

$$-\Pr\Big[\{A_2\cdot K_1\cdot K_2\cdot K_7\cdot R_1\cdot R_2\cdot S_2 + A_2\cdot K_1\cdot K_2\cdot K_7\cdot R_1\cdot(S_2+S_{21})\cdot[W]$$

$$+A_2\cdot K_1\cdot K_2\cdot K_7\cdot R_2\cdot(S_2+S_{22})\cdot[W] + K_1\cdot K_2\cdot R_1\cdot R_2\cdot S_2\cdot[W]\}$$

$$\cdot(\mathbf{CCS}+SK)\cdot M\Big]\Big\} \qquad \text{(A-4)}$$

$$= \Pr\,[P_4]\cdot\Big\{\Pr\,[A_2\cdot K_1\cdot K_7\cdot R_2\cdot(S_2+S_{22})]\cdot\Pr\,[(\mathbf{CCS}+SK)\cdot(M+M_{11}) + \overline{\mathbf{CCS}}\cdot\overline{SK}\cdot M_{21}]$$

$$+\Pr\,[K_2\cdot R_1\cdot(S_2+S_{21})]\cdot\Pr\{[W]\cdot[(\mathbf{CCS}+SK)\cdot(M+M_{11}) + \overline{\mathbf{CCS}}\cdot\overline{SK}\cdot M_{21}]\}$$

$$-\Pr\,[A_2\cdot K_1\cdot K_2\cdot K_7\cdot R_1\cdot R_2\cdot S_2]\cdot\Pr\{[W]\cdot[(\mathbf{CCS}+SK)\cdot(M+M_{11})$$

$$+\overline{\mathbf{CCS}}\cdot\overline{SK}\cdot M_{21}]\} + \Pr\,[A_2\cdot K_1\cdot K_2\cdot K_7\cdot R_1\cdot(S_2+S_{21})]\cdot\Pr\,[(\mathbf{CCS}+SK)\cdot(M+M_{12})$$

$$+\overline{\mathbf{CCS}}\cdot\overline{SK}\cdot M_{22}] + \Pr\,[K_1\cdot K_2\cdot R_2\cdot(S_2+S_{22})]\cdot\Pr\{[W]\cdot[(\mathbf{CCS}+SK)\cdot(M+M_{12})$$

$$+\overline{\mathbf{CCS}}\cdot\overline{SK}\cdot M_{22}]\} - \Pr\,[A_2\cdot K_1\cdot K_2\cdot K_7\cdot R_1\cdot R_2\cdot S_2]\cdot\Pr\{[W]\cdot[(\mathbf{CCS}+SK)$$

$$\cdot(M+M_{12}) + \overline{\mathbf{CCS}}\cdot\overline{SK}\cdot M_{22}]\} - \Pr\,[A_2\cdot K_1\cdot K_2\cdot K_7\cdot R_1\cdot R_2\cdot S_2\cdot M\cdot(\mathbf{CCS}+SK)]$$

$$-\Pr\,[A_2\cdot K_1\cdot K_2\cdot K_7\cdot R_1\cdot(S_2+S_{21}) + A_2\cdot K_1\cdot K_2\cdot K_7\cdot R_2\cdot(S_2+S_{22})$$

$$+K_1\cdot K_2\cdot R_1\cdot R_2\cdot S_2]\cdot\Pr\,[M]\cdot\Pr\{[W]\cdot(\mathbf{CCS}+SK)\} + \Pr\,[A_2\cdot K_1\cdot K_2\cdot K_7$$

$$\cdot R_1\cdot R_2\cdot S_2\cdot M]\cdot\Pr\{[W]\cdot(\mathbf{CCS}+SK)\}\Big\} \qquad \text{(A-5)}$$

$$\begin{aligned}
\Pr[S_W] = \Pr[P_4]\cdot\Big\{ & \Pr[A_2\cdot K_1\cdot K_7\cdot R_2\cdot(S_2+S_{22})]\cdot\Pr[(\mathbf{CCS}+SK)\cdot(M+M_{11})+\overline{\mathbf{CCS}}\cdot\overline{SK}\cdot M_{21}] \\
& +\Pr[K_2\cdot R_1]\cdot\{\Pr[S_2+S_{21}]-\Pr[A_2\cdot K_1\cdot K_7\cdot R_2\cdot S_2]\}\cdot\{\Pr[(\mathbf{AQ})\cdot\mathbf{Ap} \\
& \cdot A_{HG}\cdot K_6+A_1]\cdot\Pr[M+M_{11}]\cdot\Pr[\mathbf{CCS}+SK]+\Pr[A_1\cdot K_{32}\cdot M_{21}\cdot\overline{SK}] \\
& \cdot\Pr[\overline{\mathbf{CCS}}]\}\cdot\Pr[K_3]+\Pr[A_2\cdot K_1\cdot K_2\cdot K_7\cdot R_1\cdot(S_2+S_{21})]\cdot\Pr[(\mathbf{CCS}+SK) \\
& \cdot(M+M_{12})+\overline{\mathbf{CCS}}\cdot\overline{SK}\cdot M_{22}]+\Pr[K_1\cdot K_2\cdot R_2]\{\Pr[S_2+S_{22}]-\Pr[A_2\cdot K_7 \\
& \cdot R_1\cdot S_2]\}\cdot\{\Pr[(\mathbf{AQ})\cdot\mathbf{Ap}\cdot A_{HG}\cdot K_6+A_1]\cdot\Pr[M+M_{12}]\cdot\Pr[\mathbf{CCS}+SK] \\
& +\Pr[A_1\cdot K_{32}\cdot M_{22}\cdot\overline{SK}]\cdot\Pr[\overline{\mathbf{CCS}}]\}\cdot\Pr[K_3]-\Pr[A_2\cdot K_1\cdot K_2\cdot K_7\cdot R_1\cdot R_2 \\
& \cdot S_2\cdot M\cdot(\mathbf{CCS}+SK)]-\{\Pr[K_1\cdot K_2]\cdot\Pr[A_2\cdot K_7\cdot R_1\cdot(S_2+S_{21})+A_2\cdot K_7\cdot R_2 \\
& \cdot(S_2+S_{22})+R_1\cdot R_2\cdot S_2]-\Pr[A_2\cdot K_1\cdot K_2\cdot K_7\cdot R_1\cdot R_2\cdot S_2]\}\cdot\Pr[M]\cdot\Pr[K_3] \\
& \cdot\Pr[(\mathbf{AQ})\cdot\mathbf{Ap}\cdot A_{HG}\cdot K_6+A_1]\cdot\Pr[\mathbf{CCS}+SK]\Big\}
\end{aligned} \tag{A-6}$$

APPENDIX B. Derivation of Pr $[S_X \cdot T]$ for the Two-Way Doppler Function

For the two-way doppler function,

$$S_X \cdot T = P_4 \cdot \Big\{ [R_2 \cdot K_1 \cdot (S_2 + S_{22}) \cdot U + R_1 \cdot K_2 \cdot (S_2 + S_{21}) \cdot V] \cdot [(\mathbf{CCS} + C_D') \cdot (M + M_{11}) + \overline{\mathbf{CCS}} \cdot \overline{C_D'} \cdot M_{21}]$$

$$+ K_1 \cdot K_2 \; [R_1 \cdot (S_2 + S_{21}) \cdot U + R_2 \cdot (S_2 + S_{22}) \cdot V] \cdot [(\mathbf{CCS} + C_D') \cdot (M + M_{12})$$

$$+ \overline{\mathbf{CCS}} \cdot \overline{C_D'} \cdot M_{22}] \Big\} \cdot (T) \tag{B-1}$$

where

$$T = \{C_D' \cdot (G_7 + G_8) \cdot (G_9 + G_{10}) + \overline{C_D'} \cdot [(S_3 \cdot K_4)_1 \cdot G_7 + (S_3 \cdot K_4)_2 \cdot G_8] \cdot [S_{41} \cdot G_9 + S_{42} \cdot G_{10}]\}$$

$$\cdot\, G_9' \cdot G_{10}' \cdot K_4 \cdot P_7 \cdot S_3 \cdot S_4 \tag{B-2}$$

$$U = \Big[K_5 \cdot [(\mathbf{CCS} + C_D') + \overline{\mathbf{CCS}} \cdot \overline{C_D'} \cdot K_{51}] + K_5 \cdot [(\mathbf{CCS} + C_D') + \overline{\mathbf{CCS}} \cdot \overline{C_D'} \cdot K_{52}] \cdot \{(\mathbf{AQ}) \cdot \mathbf{Ap} \cdot A_{HG} \cdot K_6$$

$$\cdot\, (\mathbf{CCS} + C_D') + A_1 \cdot K_3 \cdot K_6 \cdot [(\mathbf{CCS} + C_D') + \overline{\mathbf{CCS}} \cdot \overline{C_D'} \cdot K_{32} \cdot K_{62}]\}\Big] \cdot A_2 \cdot K_7 = A_2 \cdot K_5 \cdot K_7$$

$$\cdot\, [(\mathbf{CCS} + C_D') + (A_1 \cdot K_{32} \cdot K_{52} \cdot K_{62} \cdot K_3 \cdot K_6 + K_{51}) \cdot \overline{\mathbf{CCS}} \cdot \overline{C_D'}] \tag{B-3}$$

$$V = \Big[K_5 \cdot [(\mathbf{CCS} + C_D') + \overline{\mathbf{CCS}} \cdot \overline{C_D'} \cdot K_{51}] \cdot A_2 \cdot K_7 \cdot \{(\mathbf{AQ}) \cdot \mathbf{Ap} \cdot A_{HG} \cdot K_3 \cdot K_6 \cdot (\mathbf{CCS} + C_D') + A_1 \cdot K_3$$

$$\cdot\, [(\mathbf{CCS} + C_D') + \overline{\mathbf{CCS}} \cdot \overline{C_D'} \cdot K_{32}]\} + K_5 \cdot [(\mathbf{CCS} + C_D') + \overline{\mathbf{CCS}} \cdot \overline{C_D'} \cdot K_{52}] \cdot \{(\mathbf{AQ}) \cdot \mathbf{Ap} \cdot A_{HG}$$

$$\cdot\, K_3 \cdot K_6 \cdot (\mathbf{CCS} + C_D') + A_1 \cdot K_3 \cdot K_6 \cdot [(\mathbf{CCS} + C_D') + \overline{\mathbf{CCS}} \cdot \overline{C_D'} \cdot K_{32} \cdot K_{62}]\}\Big] = K_3 \cdot K_5 \cdot \{(A_1 \cdot A_2$$

$$\cdot\, K_7 + A_1 \cdot K_6 + (\mathbf{AQ}) \cdot \mathbf{Ap} \cdot A_{HG} \cdot K_6) \cdot (\mathbf{CCS} + C_D') + A_1 \cdot K_{32} \cdot (K_{51} + K_{52} \cdot K_6 \cdot K_{62}) \cdot \overline{\mathbf{CCS}} \cdot \overline{C_D'}\}$$

$$\tag{B-4}$$

$$U \cdot V = A_2 \cdot K_3 \cdot K_5 \cdot K_7 \; [(A_1 \cdot A_2 \cdot K_7 + A_1 \cdot K_6 + (\mathbf{AQ}) \cdot \mathbf{Ap} \cdot A_{HG} \cdot K_6) \cdot (\mathbf{CCS} + C_D') + A_1 \cdot K_{32}$$

$$\cdot (K_{51} + K_{52} \cdot K_6 \cdot K_{62}) \cdot \overline{\mathbf{CCS}} \cdot \overline{C_D'}] = A_2 \cdot K_3 \cdot K_5 \cdot K_7 \cdot \; [(A_1 + (\mathbf{AQ}) \cdot \mathbf{Ap} \cdot A_{HG} \cdot K_6) \cdot (\mathbf{CCS} + C_D')$$

$$+ A_1 \cdot K_{32} \cdot (K_{51} + K_{52} \cdot K_6 \cdot K_{62}) \cdot \overline{\mathbf{CCS}} \cdot \overline{C_D'}] \qquad \text{(B-5)}$$

The associated probability is

$$\Pr\,[S_X \cdot T] = \Pr\,[P_4] \cdot \Pr\,\{[R_2 \cdot K_1 \cdot (S_2 + S_{22}) \cdot U + R_1 \cdot K_2 \cdot (S_2 + S_{21}) \cdot V] \cdot [(M + M_{11}) \cdot (\mathbf{CCS} + C_D')$$

$$+ \overline{\mathbf{CCS}} \cdot \overline{C_D'} \cdot M_{21}] \cdot [T]\} + \Pr\,[K_1 \cdot K_2] \cdot \Pr\,\{[R_1 \cdot (S_2 + S_{21}) \cdot U + R_2 \cdot K_2 \cdot (S_2 + S_{22}) \cdot V]$$

$$\cdot \; [(M + M_{12}) \cdot (\mathbf{CCS} + C_D') + \overline{\mathbf{CCS}} \cdot \overline{C_D'} \cdot M_{22}] \cdot [T]\} - \Pr\,[K_1 \cdot K_2] \cdot \Pr\,\{[R_1 \cdot R_2 \cdot S_2 \cdot U$$

$$+ R_1 \cdot (S_2 + S_{21}) \cdot U \cdot V + R_2 \cdot (S_2 + S_{22}) \cdot U \cdot V + R_1 \cdot R_2 \cdot S_2 \cdot V] \cdot \; [(\mathbf{CCS} + C_D') \cdot M] \cdot [T]\}$$

(B-6)

$$\Pr\,[S_X \cdot T] = \Pr\,[P_4] \cdot \Pr\,[G_9' \cdot G_{10}' \cdot K_4 \cdot P_7 \cdot S_3 \cdot S_4] \cdot \Pr\,\{[R_2 \cdot K_1 \cdot (S_2 + S_{22}) \cdot U + R_1 \cdot K_2 \cdot (S_2 + S_{21})$$

$$\cdot V] \cdot [(M + M_{11}) \cdot (\mathbf{CCS} + C_D')] \cdot [T']\} + \Pr\,\{[R_2 \cdot K_1 \cdot (S_2 + S_{22}) \cdot U + R_1 \cdot K_2$$

$$\cdot (S_2 + S_{21}) \cdot V] \cdot [\overline{\mathbf{CCS}} \cdot \overline{C_D'} \cdot M_{21}] \cdot [T']\} + \Pr\,[K_1 \cdot K_2] \cdot \Pr\,\{[R_1 \cdot (S_2 + S_{21}) \cdot U + R_2$$

$$\cdot (S_2 + S_{22}) \cdot V] \cdot [(M + M_{12}) \cdot (\mathbf{CCS} + C_D')] \cdot [T']\} + \Pr\,[K_1 \cdot K_2] \cdot \Pr\,\{[R_1 \cdot (S_2 + S_{21})$$

$$\cdot U + R_2 \cdot (S_2 + S_{22}) \cdot V] \cdot [\overline{\mathbf{CCS}} \cdot \overline{C_D'} \cdot M_{22}] \cdot [T']\} - \Pr\,[K_1 \cdot K_2] \cdot \Pr\,\{[R_1 \cdot R_2 \cdot S_2 \cdot U$$

$$+ R_1 \cdot (S_2 + S_{21}) \cdot U \cdot V + R_2 \cdot (S_2 + S_{22}) \cdot U \cdot V + R_1 \cdot R_2 \cdot S_2 \cdot V] \cdot [(\mathbf{CCS} + C_D') \cdot M] \cdot [T']\}$$

(B-7)

where

$$T' = \{(G_7 + G_8)\cdot(G_9 + G_{10})\cdot C_D' + [(S_3\cdot K_4)_1\cdot G_7 + (S_3\cdot K_4)_2\cdot G_8]\cdot[S_{41}\cdot G_9 + S_{42}\cdot G_{10}]\cdot \overline{C_D'}\} \tag{B-8}$$

$$= F_1\cdot C_D' + F_2\cdot \overline{C_D'} \tag{B-9}$$

$$\begin{aligned}
\Pr[S_X\cdot T] = {} & \Pr[P_4\cdot G_9'\cdot G_{10}'\cdot K_4\cdot P_7\cdot S_3\cdot S_4]\cdot\Big\{\Pr\{[R_2\cdot K_1\cdot(S_2 + S_{22})\cdot U]\cdot[(M + M_{11}) \\
& \cdot(\mathbf{CCS} + C_D')]\cdot[T']\} + \Pr\{[R_1\cdot K_2\cdot(S_2 + S_{21})\cdot V]\cdot[(M + M_{11})\cdot(\mathbf{CCS} + C_D')]\cdot[T']\} \\
& - \Pr\{[R_1\cdot R_2\cdot K_1\cdot K_2\cdot S_2\cdot U\cdot V]\cdot[(M + M_{11})\cdot(\mathbf{CCS} + C_D')]\cdot[T']\} + \Pr\{[R_2\cdot K_1 \\
& \cdot(S_2 + S_{22})\cdot U]\cdot[\overline{\mathbf{CCS}}\cdot\overline{C_D'}\cdot M_{21}]\cdot[T']\} + \Pr\{[R_1\cdot K_2\cdot(S_2 + S_{21})\cdot V]\cdot[\overline{\mathbf{CCS}}\cdot\overline{C_D'}\cdot M_{21}] \\
& \cdot[T']\} - \Pr\{[R_1\cdot R_2\cdot K_1\cdot K_2\cdot S_2\cdot U\cdot V]\cdot[\overline{\mathbf{CCS}}\cdot\overline{C_D'}\cdot M_{21}]\cdot[T']\} + \Pr[K_1\cdot K_2] \\
& \cdot\Big[\Pr\{[R_1\cdot(S_2 + S_{21})\cdot U]\cdot[(M + M_{12})\cdot(\mathbf{CCS} + C_D')]\cdot[T']\} + \Pr\{[R_2\cdot(S_2 + S_{22})\cdot V] \\
& \cdot[(M + M_{12})\cdot(\mathbf{CCS} + C_D')]\cdot[T']\} - \Pr\{[R_1\cdot R_2\cdot S_2\cdot U\cdot V]\cdot[(M + M_{12})\cdot(\mathbf{CCS} + C_D')] \\
& \cdot[T']\}\Big] + \Pr[K_1\cdot K_2]\cdot\Big[\Pr\{[R_1\cdot(S_2 + S_{21})\cdot U]\cdot[\overline{\mathbf{CCS}}\cdot\overline{C_D'}\cdot M_{22}]\cdot[T']\} \\
& + \Pr\{[R_2\cdot(S_2 + S_{22})\cdot V]\cdot[\overline{\mathbf{CCS}}\cdot\overline{C_D'}\cdot M_{22}]\cdot[T']\} - \Pr\{[R_1\cdot R_2\cdot S_2\cdot U\cdot V] \\
& \cdot[\overline{\mathbf{CCS}}\cdot\overline{C_D'}\cdot M_{22}]\cdot[T']\}\Big] - \Pr[K_1\cdot K_2]\cdot\Big[\Pr\{[R_1\cdot R_2\cdot S_2\cdot U + \Big(R_1\cdot(S_2 + S_{21}) \\
& + R_2\cdot(S_2 + S_{22})\Big)\cdot U\cdot V + R_1\cdot R_2\cdot S_2\cdot V]\cdot[(\mathbf{CCS} + C_D')\cdot M]\cdot[T']\}\Big]\Big\}
\end{aligned} \tag{B-10}$$

$$
\begin{aligned}
\Pr\,[S_X \cdot \mathrm{T}] = {} & \Pr\,[P_4 \cdot G_9' \cdot G_{10}' \cdot K_4 \cdot P_7 \cdot S_3 \cdot S_4] \cdot \Big\{ \Pr\,\{R_2 \cdot K_1 \cdot (S_2 + S_{22}) \cdot (M + M_{11}) \cdot A_2 \cdot K_5 \cdot K_7\} \\
& \cdot \Pr\,\{\,[\mathbf{CCS} + C_D'] \cdot [\mathrm{T}']\,\} + \Pr\,\{R_1 \cdot K_2 \cdot (S_2 + S_{21}) \cdot (M + M_{11}) \cdot K_3 \cdot K_5 \cdot (A_1 \cdot A_2 \cdot K_7 \\
& + A_1 \cdot K_6 + \mathbf{AQ} \cdot \mathbf{Ap} \cdot A_{HG} \cdot K_6)\} \cdot \Pr\,\{\,[\mathbf{CCS} + C_D'] \cdot [\mathrm{T}']\,\} - \Pr\,\{R_1 \cdot R_2 \cdot K_1 \cdot K_2 \cdot S_2 \\
& \cdot (M + M_{11}) \cdot A_2 \cdot K_3 \cdot K_5 \cdot K_7 \cdot (A_1 + \mathbf{AQ} \cdot \mathbf{Ap} \cdot A_{HG} \cdot K_6)\} \cdot \Pr\,\{\,[\mathbf{CCS} + C_D'] \cdot [\mathrm{T}']\,\} \\
& + \Pr\,\{R_2 \cdot K_1 \cdot (S_2 + S_{22}) \cdot M_{21} \cdot A_2 \cdot K_5 \cdot K_7 \cdot (A_1 \cdot K_{32} \cdot K_{52} \cdot K_{62} \cdot K_3 \cdot K_6 + K_{51})\} \\
& \cdot \Pr\,\{\,[\overline{\mathbf{CCS}} \cdot \overline{C_D'}] \cdot [\mathrm{T}']\,\} + \Pr\,\{R_1 \cdot K_2 \cdot (S_2 + S_{21}) \cdot M_{21} \cdot A_1 \cdot K_3 \cdot K_{32} \cdot K_5 \cdot (K_{52} \cdot K_6 \cdot K_{62} \\
& + K_{51})\} \cdot \Pr\,\{\,[\overline{\mathbf{CCS}} \cdot \overline{C_D'}] \cdot [\mathrm{T}']\,\} - \Pr\,\{R_1 \cdot R_2 \cdot K_1 \cdot K_2 \cdot S_2 \cdot M_{21} \cdot A_1 \cdot A_2 \cdot K_3 \cdot K_{32} \cdot K_5 \\
& \cdot K_7 \cdot (K_{52} \cdot K_6 \cdot K_{62} + K_{51})\} \cdot \Pr\,\{\,[\overline{\mathbf{CCS}} \cdot \overline{C_D'}] \cdot [\mathrm{T}']\,\} + \Pr\,[K_1 \cdot K_2] \cdot \Big[\Pr\,\{R_1 \cdot (S_2 + S_{21}) \\
& \cdot (M + M_{12}) \cdot A_2 \cdot K_5 \cdot K_7\} \cdot \Pr\,\{\,[\mathbf{CCS} + C_D'] \cdot [\mathrm{T}']\,\} + \Pr\,\{R_2 \cdot (S_2 + S_{22}) \cdot (M + M_{12}) \\
& \cdot K_3 \cdot K_5 \cdot (A_1 \cdot A_2 \cdot K_7 + A_1 \cdot K_6 + \mathbf{AQ} \cdot \mathbf{Ap} \cdot A_{HG} \cdot K_6)\} \cdot \Pr\,\{\,[\mathbf{CCS} + C_D'] \cdot [\mathrm{T}']\,\} \\
& - \Pr\,\{R_1 \cdot R_2 \cdot S_2 \cdot (M + M_{12}) \cdot A_2 \cdot K_3 \cdot K_5 \cdot K_7 \cdot (A_1 + \mathbf{AQ} \cdot \mathbf{Ap} \cdot A_{HG} \cdot K_6)\} \cdot \Pr\,\{\,[\mathbf{CCS} + C_D'] \\
& \cdot [\mathrm{T}']\,\} \Big] + \Pr\,[K_1 \cdot K_2] \cdot \Big[\Pr\,\{R_1 (S_2 + S_{21}) \cdot M_{22} \cdot A_2 \cdot K_5 \cdot K_7 \cdot (A_1 \cdot K_{32} \cdot K_{52} \cdot K_{62} \cdot K_3 \\
& \cdot K_6 + K_{51})\} \cdot \Pr\,\{\,[\overline{\mathbf{CCS}} \cdot \overline{C_D'}] \cdot [\mathrm{T}']\,\} + \Pr\,\{R_2 \cdot (S_2 + S_{22}) \cdot M_{22} \cdot A_1 \cdot K_3 \cdot K_{32} \cdot K_5 \\
& \cdot (K_{52} \cdot K_6 \cdot K_{62} + K_{51})\} \cdot \Pr\,\{\,[\overline{\mathbf{CCS}} \cdot \overline{C_D'}] \cdot [\mathrm{T}']\,\} - \Pr\,\{R_1 \cdot R_2 \cdot S_2 \cdot M_{22} \cdot A_1 \cdot A_2 \cdot K_3 \cdot K_{32} \\
& \cdot K_5 \cdot K_7 (K_{52} \cdot K_6 \cdot K_{62} + K_{51})\} \cdot \Pr\,\{\,[\overline{\mathbf{CCS}} \cdot \overline{C_D'}] \cdot [\mathrm{T}']\,\} \Big] - \Pr\,[K_1 \cdot K_2] \cdot \Big[\Pr\,\{R_1 \cdot R_2 \\
& \cdot S_2 \cdot A_2 \cdot K_5 \cdot K_7 \cdot M\} \cdot \Pr\,\{\,[\mathbf{CCS} + C_D'] \cdot [\mathrm{T}']\,\} + \Pr\,\{R_1 \cdot R_2 \cdot S_2 \cdot M \cdot K_3 \cdot K_5 \cdot (A_1 \cdot A_2 \cdot K_7 + A_1 \\
& \cdot K_6 + \mathbf{AQ} \cdot \mathbf{Ap} \cdot A_{HG} \cdot K_6)\} \cdot \Pr\,\{\,[\mathbf{CCS} + C_D'] \cdot [\mathrm{T}']\,\} + \{\Pr\,[R_1 \cdot (S_2 + S_{21}) + R_2 \cdot (S_2 + S_{22})] \\
& - 2\Pr\,[R_1 \cdot R_2 \cdot S_2]\} \cdot \Pr\,\{A_2 \cdot K_3 \cdot K_5 \cdot K_7 \cdot M \cdot (A_1 + \mathbf{AQ} \cdot \mathbf{Ap} \cdot A_{HG} \cdot K_6)\} \cdot \Pr\,\{\,[\mathbf{CCS} + C_D'] \cdot [\mathrm{T}']\,\} \Big] \Big\}
\end{aligned}
\tag{B-11}
$$

APPENDIX C. Derivation of Pr $[D_O]$ for the One-Way Doppler Function

For the one-way doppler function,

$$D_O = (\mathbf{PO})\cdot(\mathbf{AC})\cdot K_4\cdot K_5\cdot P_4\cdot P_7\cdot S_3\cdot S_4\cdot [G_7\cdot G_{11}\cdot (S_3\cdot K_4)_1 + G_8\cdot G_{12}\cdot (S_3\cdot K_4)_2]\cdot [G_9\cdot S_{41} + G_{10}\cdot S_{42}]$$

$$\cdot \{[A_2\cdot K_7 + \mathbf{AQ}\cdot \mathbf{Ap}\cdot A_{HG}\cdot K_6 + A_1\cdot K_3\cdot K_6]\cdot \mathbf{CCS} + [A_2\cdot K_7\cdot K_{51} + A_1\cdot K_3\cdot K_{32}\cdot K_{52}\cdot K_6$$

$$\cdot K_{62}]\cdot \overline{\mathbf{CCS}}\}\cdot G_9'\cdot G_{10}' \qquad \text{(C-1)}$$

The associated probability of success is

$$\Pr[D_O] = \Pr[(\mathbf{PO})\cdot(\mathbf{AC})]\cdot \Pr[K_4\cdot K_5\cdot P_4\cdot P_7\cdot S_3\cdot S_4]\cdot \Pr[G_7\cdot G_{11}\cdot (S_3\cdot K_4)_1 + G_8\cdot G_{12}\cdot (S_3\cdot K_4)_2]$$

$$\cdot \Pr[G_9\cdot S_{41} + G_{10}\cdot S_{42}]\cdot \Pr\{[A_2\cdot K_7 + \mathbf{AQ}\cdot \mathbf{Ap}\cdot A_{HG}\cdot K_6 + A_1\cdot K_3\cdot K_6]\cdot \mathbf{CCS}$$

$$+ [A_2\cdot K_7\cdot K_{51} + A_1\cdot K_3\cdot K_{32}\cdot K_{52}\cdot K_6\cdot K_{62}]\cdot \overline{\mathbf{CCS}}\}\cdot \Pr[G_9'\cdot G_{10}'] \qquad \text{(C-2)}$$

By letting

$$\mathrm{H} = \Pr[(\mathbf{PO})\cdot(\mathbf{AC})]\cdot \Pr[K_4\cdot K_5\cdot P_4\cdot P_7\cdot S_3\cdot S_4]\cdot \Pr[G_7\cdot G_{11}\cdot (S_3\cdot K_4)_1 + G_8\cdot G_{12}\cdot (S_3\cdot K_4)_2]$$

$$\cdot \Pr[G_9\cdot S_{41} + G_{10}\cdot S_{42}]\cdot \Pr[G_9'\cdot G_{10}'] \qquad \text{(C-3)}$$

and substituting this value into Eq. (C-2),

$$\Pr[D_O] = \mathrm{H}\Big[\Pr[A_2\cdot K_7 + \mathbf{AQ}\cdot \mathbf{Ap}\cdot A_{HG}\cdot K_6 + A_1\cdot K_3\cdot K_6]\cdot \Pr[\mathbf{CCS}] + \Pr[A_2\cdot K_7\cdot K_{51} + A_1\cdot K_3$$

$$\cdot K_{32}\cdot K_{52}\cdot K_6\cdot K_{62}]\{1 - \Pr[\mathbf{CCS}]\}\Big] \qquad \text{(C-4)}$$

$$
\begin{aligned}
\Pr[D_O] = \mathrm{H}\Big[&\Pr[A_2 \cdot K_7 \cdot K_{51} + A_1 \cdot K_3 \cdot K_{32} \cdot K_{52} \cdot K_6 \cdot K_{62} + \{\Pr[A_2 \cdot K_7] + \Pr[A_1 \cdot K_3 \cdot K_6] \\
&\cdot (1 - \Pr[A_2 \cdot K_7]) - \Pr[A_2 \cdot K_7 \cdot K_{51} + A_1 \cdot K_3 \cdot K_{32} \cdot K_{52} \cdot K_6 \cdot K_{62}]\} \cdot \Pr[\mathbf{CCS}] \\
&+ \Pr[A_{HG} \cdot K_6] \cdot (1 - \Pr[A_2 \cdot K_7]) \cdot (1 - \Pr[A_1 \cdot K_3]) \cdot \Pr[\mathbf{CCS}] \cdot \Pr[\mathbf{AQ} \cdot \mathbf{Ap}]\Big]
\end{aligned}
\tag{C-5}
$$

APPENDIX D. Derivation of Pr $[D_O \cdot D_T]$ for the One- or Two-Way Doppler Function

For the required probability

$$\Pr[D_O + D_T] = \Pr[D_O] + \Pr[D_T] - \Pr[D_O \cdot D_T] \tag{D-1}$$

the only term that still must be obtained is Pr $[D_O \cdot D_T]$.

As has been obtained,

$$\begin{aligned} D_O = {} & (\mathbf{PO})\cdot(\mathbf{AC})\cdot G_9' \cdot G_{10}' \cdot K_4 \cdot K_5 \cdot P_4 \cdot P_7 \cdot S_3 \cdot S_4 \cdot [G_7 \cdot G_{11} \cdot (S_3 \cdot K_4)_1 + G_8 \cdot G_{12} \cdot (S_3 \cdot K_4)_2] \\ & \cdot [G_9 \cdot S_{41} + G_{10} \cdot S_{42}] \cdot [(A_2 \cdot K_7 + \mathbf{AQ}\cdot\mathbf{Ap}\cdot A_{HG} \cdot K_6 + A_1 \cdot K_3 \cdot K_6)\cdot \mathbf{CCS} + (A_2 \cdot K_7 \cdot K_{51} \\ & + A_1 \cdot K_3 \cdot K_{32} \cdot K_{52} \cdot K_6 \cdot K_{62}) \cdot \overline{\mathbf{CCS}}] \end{aligned} \tag{D-2}$$

$$\begin{aligned} D_T = {} & (\mathbf{PO})\cdot(\mathbf{AC})\cdot G_9' \cdot G_{10}' \cdot K_4 \cdot P_4 \cdot P_7 \cdot S_3 \cdot S_4 \cdot \{[C_D' + \overline{C_D'} \cdot (S_3 \cdot K_4)_1] \cdot G_7 + [C_D' + \overline{C_D'} \cdot (S_3 \cdot K_4)_2] \cdot G_8\} \\ & \cdot \{[C_D' + \overline{C_D'} \cdot S_{41}] \cdot G_9 + [C_D' + \overline{C_D'} \cdot S_{42}] \cdot G_{10}\} \cdot \{[R_2 \cdot K_1 \cdot (S_2 + S_{22}) \cdot \mathrm{U} + R_1 \cdot K_2 \cdot (S_2 + S_{21}) \cdot \mathrm{V}] \\ & \cdot [(\mathbf{CCS} + C_D') \cdot (M + M_{11}) + \overline{\mathbf{CCS}} \cdot \overline{C_D'} \cdot M_{21}] + [R_1 \cdot K_1 \cdot K_2 \cdot (S_2 + S_{21}) \cdot \mathrm{U} + R_2 \cdot K_1 \cdot K_2 \cdot (S_2 + S_{22}) \cdot \mathrm{V}] \\ & \cdot [(\mathbf{CCS} + C_D') \cdot (M + M_{12}) + \overline{\mathbf{CCS}} \cdot \overline{C_D'} \cdot M_{22}]\} \end{aligned} \tag{D-3}$$

where

$$\mathrm{U} = A_2 \cdot K_5 \cdot K_7 \cdot [(\mathbf{CCS} + C_D') + (A_1 \cdot K_{32} \cdot K_{52} \cdot K_{62} \cdot K_3 \cdot K_6 + K_{51}) \cdot \overline{\mathbf{CCS}} \cdot \overline{C_D'}] \tag{D-4}$$

$$\begin{aligned} \mathrm{V} = {} & K_3 \cdot K_5 \cdot [(A_1 \cdot A_2 \cdot K_7 + A_1 \cdot K_6 + \mathbf{AQ}\cdot\mathbf{Ap} \cdot A_{HG} \cdot K_6) \cdot (\mathbf{CCS} + C_D') + (A_1 \cdot K_{32}) \cdot (K_{51} + K_{52} \cdot K_6 \cdot K_{62}) \\ & \cdot \overline{\mathbf{CCS}} \cdot \overline{C_D'}] \end{aligned} \tag{D-5}$$

$$U \cdot V = A_2 \cdot K_3 \cdot K_5 \cdot K_7 \cdot [(A_1 + \mathbf{AQ} \cdot \mathbf{Ap} \cdot A_{HG} \cdot K_6) \cdot (\mathbf{CCS} + C_D') + A_1 \cdot K_{32} \cdot (K_{51} + K_{52} \cdot K_6 \cdot K_{62}) \cdot \overline{\mathbf{CCS}} \cdot \overline{C_D'}] \tag{D-6}$$

By using Eq. (D-2) and (D-3),

$$D_O \cdot D_T = (\mathbf{PO}) \cdot (\mathbf{AC}) \cdot G_9' \cdot G_{10}' \cdot K_4 \cdot K_5 \cdot P_4 \cdot P_7 \cdot S_3 \cdot S_4 \cdot \Big[[G_7 \cdot G_{11} \cdot (S_3 \cdot K_4)_1 + G_8 \cdot G_{12} \cdot (S_3 \cdot K_4)_2] \cdot \{ [C_D' + \overline{C_D'} (S_3 \cdot K_4)_1] \cdot G_7 + [C_D' + \overline{C_D'} \cdot (S_3 \cdot K_4)_2] \cdot G_8 \} \Big] \cdot \Big[[G_9 \cdot S_{41} + G_{10} \cdot S_{42}] \cdot \{ [C_D' + \overline{C_D'} \cdot S_{41}] \cdot G_9 + [C_D' + \overline{C_D'} \cdot S_{42}] \cdot G_{10} \} \Big] \cdot \Big[(A_2 \cdot K_7 + \mathbf{AQ} \cdot \mathbf{Ap} \cdot A_{HG} \cdot K_6 + A_1 \cdot K_3 \cdot K_6) \cdot \mathbf{CCS} + (A_2 \cdot K_7 \cdot K_{51} + A_1 \cdot K_3 \cdot K_{32} \cdot K_{52} \cdot K_6 \cdot K_{62}) \cdot \overline{\mathbf{CCS}} \Big] \cdot \Big[[R_2 \cdot K_1 \cdot (S_2 + S_{22}) \cdot U + R_1 \cdot K_2 \cdot (S_2 + S_{21}) \cdot V] \cdot [(\mathbf{CCS} + C_D') \cdot (M + M_{11}) + \overline{\mathbf{CCS}} \cdot \overline{C_D'} \cdot M_{21}] + [R_1 \cdot K_1 \cdot K_2 \cdot (S_2 + S_{21}) \cdot U + R_2 \cdot K_1 \cdot K_2 \cdot (S_2 + S_{22}) \cdot V] \cdot [\mathbf{CCS} + C_D') \cdot (M + M_{12}) + \overline{\mathbf{CCS}} \cdot \overline{C_D'} \cdot M_{22}] \Big] \tag{D-7}$$

Let

$$W' = [(A_2 \cdot K_7 + \mathbf{AQ} \cdot \mathbf{Ap} \cdot A_{HG} \cdot K_6 + A_1 \cdot K_3 \cdot K_6) \cdot \mathbf{CCS} + (A_2 \cdot K_7 \cdot K_{51} + A_1 \cdot K_3 \cdot K_{32} \cdot K_{52} \cdot K_6 \cdot K_{62}) \cdot \overline{\mathbf{CCS}}] \tag{D-8}$$

$$U' = A_2 \cdot K_7 \cdot [(\mathbf{CCS} + C_D') + (A_1 \cdot K_{32} \cdot K_{52} \cdot K_{62} \cdot K_3 \cdot K_6 + K_{51}) \cdot \overline{\mathbf{CCS}} \cdot \overline{C_D'}] \tag{D-9}$$

$$V' = K_3 \cdot [(A_1 \cdot A_2 \cdot K_7 + A_1 \cdot K_6 + \mathbf{AQ} \cdot \mathbf{Ap} \cdot A_{HG} \cdot K_6) \cdot (\mathbf{CCS} + C_D') + A_1 \cdot K_{32} \cdot (K_{51} + K_{52} \cdot K_6 \cdot K_{62}) \cdot \overline{\mathbf{CCS}} \cdot \overline{C_D'}] \tag{D-10}$$

Rearranging Eq. (D-7),

$$D_O \cdot D_T = (\mathbf{PO}) \cdot (\mathbf{AC}) \cdot G_9' \cdot G_{10}' \cdot K_4 \cdot K_5 \cdot P_4 \cdot P_7 \cdot S_3 \cdot S_4 \cdot [G_7 \cdot G_{11} \cdot (S_3 \cdot K_4)_1 + G_8 \cdot G_{12} \cdot (S_3 \cdot K_4)_2]$$
$$\cdot [G_9 \cdot S_{41} + G_{10} \cdot S_{42}] \cdot \{ [R_2 \cdot K_1 \cdot (S_2 + S_{22}) \cdot W' \cdot U' + R_1 \cdot K_2 \cdot (S_2 + S_{21}) \cdot W' \cdot V']$$
$$\cdot [(\mathbf{CCS} + C_D') \cdot (M + M_{11}) + \overline{\mathbf{CCS}} \cdot \overline{C_D'} \cdot M_{21}] + [R_1 \cdot K_1 \cdot K_2 \cdot (S_2 + S_{21}) \cdot W' \cdot U' + R_2 \cdot K_1 \cdot K_2$$
$$\cdot (S_2 + S_{22}) \cdot W \cdot V'] \cdot [(\mathbf{CCS} + C_D') \cdot (M + M_{12}) + \overline{\mathbf{CCS}} \cdot \overline{C_D'} \cdot M_{22}] \} \tag{D-11}$$

The associated probability is

$$\Pr [D_O \cdot D_T] = \Pr \{(\mathbf{PO}) \cdot (\mathbf{AC}) \cdot G_9' \cdot G_{10}' \cdot K_4 \cdot K_5 \cdot P_4 \cdot P_7 \cdot S_3 \cdot S_4 \cdot [G_7 \cdot G_{11} \cdot (S_3 \cdot K_4)_1 + G_8 \cdot G_{12}$$
$$\cdot (S_3 \cdot K_4)_2] \cdot (G_9 \cdot S_{41} + G_{10} \cdot S_{42}] \} \cdot \Pr \{ [R_2 \cdot K_1 \cdot (S_2 + S_{22}) \cdot W' \cdot U' + R_1 \cdot K_2$$
$$\cdot (S_2 + S_{21}) \cdot W' \cdot V'] \cdot [(\mathbf{CCS} + C_D') \cdot (M + M_{11}) + \overline{\mathbf{CCS}} \cdot \overline{C_D'} \cdot M_{21}] + [R_1 \cdot K_1 \cdot K_2$$
$$\cdot (S_2 + S_{21}) \cdot W' \cdot U' + R_2 \cdot K_1 \cdot K_2 \cdot (S_2 + S_{22}) \cdot W' \cdot V'] \cdot [(\mathbf{CCS} + C_D') \cdot (M + M_{11})$$
$$+ \overline{\mathbf{CCS}} \cdot \overline{C_D'} \cdot M_{22}] \} \tag{D-12}$$

If we let

$$J_0 = \Pr \{(\mathbf{PO}) \cdot (\mathbf{AC}) \cdot G_9' \cdot G_{10}' \cdot K_4 \cdot K_5 \cdot P_4 \cdot P_7 \cdot S_3 \cdot S_4 \cdot [G_7 \cdot G_{11} \cdot (S_3 \cdot K_4)_1 + G_8 \cdot G_{12} \cdot (S_3 \cdot K_4)_2]$$
$$\cdot [G_9 \cdot S_{41} + G_{10} \cdot S_{42}] \} \tag{D-13}$$

then

$$\Pr[D_O \cdot D_T] = J_0 \cdot \Big\{ \Pr\{[R_2 \cdot K_1 \cdot (S_2 + S_{22}) \cdot W' \cdot U' + R_1 \cdot K_2 \cdot (S_2 + S_{21}) \cdot W' \cdot V'] \cdot [(\mathbf{CCS} + C_D') \cdot (M + M_{11}) + \overline{\mathbf{CCS}} \cdot \overline{C_D'} \cdot M_{21}]\} + \Pr\{[R_1 \cdot K_1 \cdot K_2 \cdot (S_2 + S_{21}) \cdot W' \cdot U' \cdot R_2 \cdot K_1 \cdot K_2 \cdot (S_2 + S_{22}) \cdot W' \cdot V'] \cdot [(\mathbf{CCS} + C_D') \cdot (M + M_{12}) + \overline{\mathbf{CCS}} \cdot \overline{C_D'} \cdot M_{22}]\} - \Pr\{[R_1 \cdot R_2 \cdot K_1 \cdot K_2 \cdot S_2 \cdot W' \cdot U' + R_2 \cdot K_1 \cdot K_2 \cdot (S_2 + S_{22}) \cdot W' \cdot U' \cdot V' + R_1 \cdot K_1 \cdot K_2 \cdot (S_2 + S_{21}) \cdot W' \cdot U' \cdot V' + R_1 \cdot R_2 \cdot K_1 \cdot K_2 \cdot S_2 \cdot W' \cdot V'] \cdot [(\mathbf{CCS} + C_D') \cdot M]\} \Big\}$$

(D-14)

$$\Pr[D_O \cdot D_T] = J_0 \cdot \Big\{ \Pr[M + M_{11}] \cdot \{\Pr[R_2 \cdot K_1 \cdot (S_2 + S_{22})] \cdot \Pr[W' \cdot U' \cdot (\mathbf{CCS} + C_D')] + \Pr[R_1 \cdot K_2 \cdot (S_2 + S_{21})] \cdot \Pr[W' \cdot V' \cdot (\mathbf{CCS} + C_D')] - \Pr[R_1 \cdot R_2 \cdot K_1 \cdot K_2 \cdot S_2] \cdot \Pr[W' \cdot U' \cdot V' \cdot (\mathbf{CCS} + C_D')]\} + \Pr[M_{21}] \cdot \{\Pr[R_2 \cdot K_1 \cdot (S_2 + S_{22})] \cdot \Pr[W' \cdot U' \cdot \overline{\mathbf{CCS}} \cdot \overline{C_D'}] + \Pr[R_1 \cdot K_2 \cdot (S_2 + S_{21}] \cdot \Pr[W' \cdot V' \cdot \overline{\mathbf{CCS}} \cdot \overline{C_D'}] - \Pr[R_1 \cdot R_2 \cdot K_1 \cdot K_2 \cdot S_2] \cdot \Pr[W' \cdot U' \cdot V' \cdot \overline{\mathbf{CCS}} \cdot \overline{C_D'}]\} + \Pr[M + M_{12}] \cdot \{\Pr[R_1 \cdot K_1 \cdot K_2 \cdot (S_2 + S_{21})] \cdot \Pr[W' \cdot U' \cdot (\mathbf{CCS} + C_D')] + \Pr[R_2 \cdot K_1 \cdot K_2 \cdot (S_2 + S_{22})] \cdot \Pr[W' \cdot V' \cdot (\mathbf{CCS} + C_D')] - \Pr[R_1 \cdot R_2 \cdot K_1 \cdot K_2 \cdot S_2] \cdot \Pr[W' \cdot U' \cdot V' \cdot (\mathbf{CCS} + C_D')]\} + \Pr[M_{22}] \cdot \{\Pr[R_1 \cdot K_1 \cdot K_2 \cdot (S_2 + S_{21})] \cdot \Pr[W' \cdot U' \cdot \overline{\mathbf{CCS}} \cdot \overline{C_D'}] + \Pr[R_2 \cdot K_1 \cdot K_2 \cdot (S_2 + S_{22})] \cdot \Pr[W' \cdot V' \cdot \overline{\mathbf{CCS}} \cdot \overline{C_D'}] - \Pr[R_1 \cdot R_2 \cdot K_1 \cdot K_2 \cdot S_2] \cdot \Pr[W' \cdot U' \cdot V' \cdot \overline{\mathbf{CCS}} \cdot \overline{C_D'}]\} - \Pr[M] \cdot \{\Pr[R_1 \cdot R_2 \cdot K_1 \cdot K_2 \cdot S_2] \cdot \Pr[W' \cdot U' \cdot (\mathbf{CCS} + C_D')] + \Pr[R_2 \cdot K_1 \cdot K_2 \cdot (S_2 + S_{22}) + R_1 \cdot K_1 \cdot K_2 \cdot (S_2 + S_{21})] \cdot \Pr[W' \cdot U' \cdot V' \cdot (\mathbf{CCS} + C_D')] + \Pr[R_1 \cdot R_2 \cdot K_1 \cdot K_2 \cdot S_2] \cdot \Pr[W' \cdot V' \cdot (\mathbf{CCS} + C_D')] - 2\Pr[R_1 \cdot R_2 \cdot K_1 \cdot K_2 \cdot S_2] \cdot \Pr[W' \cdot U' \cdot V' \cdot (\mathbf{CCS} + C_D')]\} \Big\}$$

(D-15)

where

$$\mathbf{W'} \cdot \mathbf{U'} = [(A_2 \cdot K_7 + \mathbf{AQ} \cdot \mathbf{Ap} \cdot A_{HG} \cdot K_6 + A_1 \cdot K_3 \cdot K_6) \cdot \mathbf{CCS} + (A_2 \cdot K_7 \cdot K_{51} + A_1 \cdot K_3 \cdot K_{32} \cdot K_{52} \cdot K_6 \cdot K_{62})$$

$$\cdot \overline{\mathbf{CCS}}] \cdot A_2 \cdot K_7 \cdot [(\mathbf{CCS} + C_D') + (A_1 \cdot K_{32} \cdot K_{52} \cdot K_{62} \cdot K_3 \cdot K_6 + K_{51}) \cdot \overline{\mathbf{CCS}} \cdot \overline{C_D'}] = A_2 \cdot K_7$$

$$\cdot [\mathbf{CCS} + (K_{51} + A_1 \cdot K_3 \cdot K_{52} \cdot K_6 \cdot K_{62}) \cdot \overline{\mathbf{CCS}}] \cdot [(\mathbf{CCS} + C_D') + (A_1 \cdot K_{32} \cdot K_{52} \cdot K_{62} \cdot K_3$$

$$\cdot K_6 + K_{51}) \cdot \overline{\mathbf{CCS}} \cdot \overline{C_D'}] = A_2 \cdot K_7 \cdot [\mathbf{CCS} + (K_{51} + A_1 \cdot K_3 \cdot K_{32} \cdot K_{52} \cdot K_6 \cdot K_{62}) \cdot C_D' \cdot \overline{\mathbf{CCS}}$$

$$+ (K_{51} + A_1 \cdot K_3 \cdot K_{32} \cdot K_{52} \cdot K_6 \cdot K_{62}) \cdot \overline{\mathbf{CCS}} \cdot \overline{C_D'}] = A_2 \cdot K_7 \cdot [\mathbf{CCS} + (K_{51} + A_1 \cdot K_3 \cdot K_{32}$$

$$\cdot K_{52} \cdot K_6 \cdot K_{62}) \cdot \overline{\mathbf{CCS}}] \tag{D-16}$$

$$\mathbf{W'} \cdot \mathbf{V'} = [(A_2 \cdot K_7 + \mathbf{AQ} \cdot \mathbf{Ap} \cdot A_{HG} \cdot K_6 + A_1 \cdot K_6) \cdot \mathbf{CCS} + (A_2 \cdot K_7 \cdot K_{51} + A_1 \cdot K_3 \cdot K_{32} \cdot K_{52} \cdot K_6 \cdot K_{62}) \cdot \mathbf{CCS}]$$

$$\cdot K_3 \cdot [(A_1 \cdot A_2 \cdot K_7 + A_1 \cdot K_6 + \mathbf{AQ} \cdot \mathbf{Ap} \cdot A_{HG} \cdot K_6) \cdot (\mathbf{CCS} + C_D') + A_1 \cdot K_{32} \cdot (K_{51} + K_{52} \cdot K_6 \cdot K_{62})$$

$$\cdot \overline{\mathbf{CCS}} \cdot \overline{C_D'}] = K_3 \cdot [(A_2 \cdot K_7 + \mathbf{AQ} \cdot \mathbf{Ap} \cdot A_{HG} \cdot K_6 + A_1 \cdot K_6) \cdot \mathbf{CCS} + (A_2 \cdot K_7 \cdot K_{51} + A_1 \cdot K_{32}$$

$$\cdot K_{52} \cdot K_6 \cdot K_{62}) \cdot \overline{\mathbf{CCS}}] \cdot [(A_1 \cdot A_2 \cdot K_7 + A_1 \cdot K_6 + \mathbf{AQ} \cdot \mathbf{Ap} \cdot A_{HG} \cdot K_6) \cdot (\mathbf{CCS} + C_D') + A_1 \cdot K_{32}$$

$$\cdot (K_{51} + K_{52} \cdot K_6 \cdot K_{62}) \cdot \overline{\mathbf{CCS}} \cdot \overline{C_D'}] = K_3 \cdot [(A_1 \cdot A_2 \cdot K_7 + \mathbf{AQ} \cdot \mathbf{Ap} \cdot A_{HG} \cdot K_6 + A_1 \cdot K_6) \cdot \mathbf{CCS}$$

$$+ A_1 \cdot K_{32} \cdot (A_2 \cdot K_7 \cdot K_{51} + K_{52} \cdot K_6 \cdot K_{62}) \cdot \overline{\mathbf{CCS}} \cdot \overline{C_D'} + (A_2 \cdot K_7 \cdot K_{51} + A_1 \cdot K_{32} \cdot K_{52} \cdot K_6 \cdot K_{62})$$

$$\cdot (A_1 \cdot A_2 \cdot K_7 + A_1 \cdot K_6 + \mathbf{AQ} \cdot \mathbf{Ap} \cdot A_{HG} \cdot K_6) \cdot \overline{\mathbf{CCS}} \cdot \overline{C_D'}] = K_3 \cdot [\alpha_1 \cdot \mathbf{CCS} + (\beta_1 \cdot \overline{C_D'} + \gamma_1 \cdot C_D')$$

$$\cdot \overline{\mathbf{CCS}}] \tag{D-17}$$

$$
\begin{aligned}
\mathrm{W}'\cdot\mathrm{U}'\cdot\mathrm{V}' = {} & A_2\cdot K_3\cdot K_7\cdot\ [\mathbf{CCS}+(K_{51}+A_1\cdot K_3\cdot K_{32}\cdot K_{52}\cdot K_6\cdot K_{62})\cdot\overline{\mathbf{CCS}}]\cdot[(A_1\cdot A_2\cdot K_7+\mathbf{AQ}\cdot\mathbf{Ap}\\
&\cdot A_{HG}\cdot K_6+A_1\cdot K_6)\cdot\mathbf{CCS}+A_1\cdot K_{32}\cdot(A_2\cdot K_7\cdot K_{51}+K_{52}\cdot K_6\cdot K_{62})\cdot\overline{\mathbf{CCS}}\cdot\overline{C_D'}+(A_2\cdot K_7\\
&\cdot K_{51}+A_1\cdot K_{32}\cdot K_{52}\cdot K_6\cdot K_{62})\cdot(A_1\cdot A_2\cdot K_7+A_1\cdot K_6+\mathbf{AQ}\cdot\mathbf{AP}\cdot A_{HG}\cdot K_6)\cdot\overline{\mathbf{CCS}}\cdot C_D']\\
&= A_2\cdot K_3\cdot K_7\cdot\ [\mathbf{CCS}+(K_{51}+A_1\cdot K_{32}\cdot K_{52}\cdot K_6\cdot K_{62})\cdot\overline{\mathbf{CCS}}]\cdot[(A_1+\mathbf{AQ}\cdot\mathbf{Ap}\cdot A_{HG}\cdot K_6)\\
&\cdot\mathbf{CCS}+A_1\cdot K_{32}\cdot(K_{51}+K_{52}\cdot K_6\cdot K_{62})\cdot\overline{\mathbf{CCS}}\cdot\overline{C_D'}+(K_{51}+A_1\cdot K_{32}\cdot K_{52}\cdot K_6\cdot K_{62})\\
&\cdot(A_1+\mathbf{AQ}\cdot\mathbf{Ap}\cdot A_{HG}\cdot K_6)\cdot\overline{\mathbf{CCS}}\cdot C_D'] = A_2\cdot K_3\cdot K_7\cdot[A_1+\mathbf{AQ}\cdot\mathbf{Ap}\cdot A_{HG}\cdot K_6)\cdot\mathbf{CCS}+A_1\\
&\cdot K_{32}\cdot(K_{51}+K_{52}\cdot K_6\cdot K_{62})\cdot\overline{\mathbf{CCS}}\cdot\overline{C_D'}+(K_{51}+A_1\cdot K_{32}\cdot K_{52}\cdot K_6\cdot K_{62})\cdot(A_1+\mathbf{AQ}\cdot\mathbf{Ap}\\
&\cdot A_{HG}\cdot K_6)\cdot\overline{\mathbf{CCS}}\cdot C_D'] = A_2\cdot K_3\cdot K_7\cdot[\alpha_2\cdot\mathbf{CCS}+(\beta_2\cdot\overline{C_D'}+\gamma_2\cdot C_D')\cdot\overline{\mathbf{CCS}}]
\end{aligned}
\tag{D-18}
$$

and

$$\alpha_1 = (A_1\cdot A_2\cdot K_7+\mathbf{AQ}\cdot\mathbf{Ap}\cdot A_{HG}\cdot K_6+A_1\cdot K_6) \tag{D-19}$$

$$\alpha_2 = (A_1+\mathbf{AQ}\cdot\mathbf{Ap}\cdot A_{HG}\cdot K_6) \tag{D-20}$$

$$\beta_1 = A_1\cdot K_{32}\cdot(A_2\cdot K_7\cdot K_{51}+K_{52}\cdot K_6\cdot K_{62}) \tag{D-21}$$

$$\beta_2 = A_1\cdot K_{32}\cdot(K_{51}+K_{52}\cdot K_6\cdot K_{62}) \tag{D-22}$$

$$\gamma_1 = (A_2\cdot K_7\cdot K_{51}+A_1\cdot K_{32}\cdot K_{52}\cdot K_6\cdot K_{62})\cdot(A_1\cdot A_2\cdot K_7+A_1\cdot K_6+\mathbf{AQ}\cdot\mathbf{Ap}\cdot A_{HG}\cdot K_6) \tag{D-23}$$

$$\gamma_2 = (K_{51}+A_1\cdot K_{32}\cdot K_{52}\cdot K_6\cdot K_{62})\cdot(A_1+\mathbf{AQ}\cdot\mathbf{Ap}\cdot A_{HG}\cdot K_6) \tag{D-24}$$

For numerical evaluation of Eq. (D-15), it is necessary to obtain the following six equations:

(1) $\Pr\ [\mathrm{W}' \cdot \mathrm{U}' \cdot (\mathbf{CCS} + C_D')]$

(2) $\Pr\ [\mathrm{W}' \cdot \mathrm{V}' \cdot (\mathbf{CCS} + C_D')]$

(3) $\Pr\ [\mathrm{W}' \cdot \mathrm{U}' \cdot \overline{\mathbf{CCS}} \cdot \overline{C_D'}]$

(4) $\Pr\ [\mathrm{W}' \cdot \mathrm{V}' \cdot \overline{\mathbf{CCS}} \cdot \overline{C_D'}]$

(5) $\Pr\ [\mathrm{W}' \cdot \mathrm{U}' \cdot \mathrm{V}' \cdot (\mathbf{CCS} + C_D')]$

(6) $\Pr\ [\mathrm{W}' \cdot \mathrm{U}' \cdot \mathrm{V}' \cdot \overline{\mathbf{CCS}} \cdot \overline{C_D'}]$

$$
\begin{aligned}
\Pr\ [\mathrm{W}' \cdot \mathrm{U}' \cdot (\mathbf{CCS} + C_D')] &= \Pr\ \{A_2 \cdot K_7 \cdot [\mathbf{CCS} + (K_{51} + A_1 \cdot K_3 \cdot K_{32} \cdot K_{52} \cdot K_6 \cdot K_{62}) \cdot \overline{\mathbf{CCS}}] \cdot (\mathbf{CCS} + C_D')\} \\
&= \Pr\ \{A_2 \cdot K_7 \cdot [\mathbf{CCS} + (K_{51} + A_1 \cdot K_3 \cdot K_{32} \cdot K_{52} \cdot K_6 \cdot K_{62}) \cdot C_D' \cdot \overline{\mathbf{CCS}}]\} \\
&= \Pr\ [A_2 \cdot K_7] \cdot \Big[\Pr\ [(K_{51} + A_1 \cdot K_3 \cdot K_{32} \cdot K_{52} \cdot K_6 \cdot K_{62}) \cdot C_D'] + \{1 - \Pr\ [(K_{51} \\
&\quad + A_1\ K_3 \cdot K_{32} \cdot K_{52} \cdot K_6 \cdot K_{62}) \cdot C_D']\} \cdot \Pr\ [\mathbf{CCS}]\Big] \qquad \text{(D-25)}
\end{aligned}
$$

$$
\begin{aligned}
\Pr\ [\mathrm{W}' \cdot \mathrm{V}' \cdot (\mathbf{CCS} + C_D')] &= \Pr\ \{K_3 \cdot [\alpha_1 \cdot \mathbf{CCS} + (\beta_1 \cdot \overline{C_D'} + \gamma_1 \cdot C_D') \cdot \overline{\mathbf{CCS}}] \cdot (\mathbf{CCS} + C_D')\} = \Pr\ [K_3] \\
&\cdot \Pr\ [\alpha_1 \cdot \mathbf{CCS} + \gamma_1 \cdot C_D' \cdot \overline{\mathbf{CCS}}] = \Pr\ [K_3] \cdot \{\Pr\ [A_1 \cdot A_2 \cdot K_7 + A_1 \cdot K_6 \\
&+ \mathbf{AQ} \cdot \mathbf{Ap} \cdot A_{HG} \cdot K_6] \cdot \Pr\ [\mathbf{CCS}] + \Pr\ [(A_1 \cdot A_2 \cdot K_7 + A_1 \cdot K_6 + \mathbf{AQ} \cdot \mathbf{Ap} \cdot A_{HG} \\
&\cdot K_6) \cdot (A_2 \cdot K_7 \cdot K_{51} + A_1 \cdot K_{32} \cdot K_{52} \cdot K_6 \cdot K_{62}) \cdot C_D'] - \Pr\ [(A_1 \cdot A_2 \cdot K_7 + A_1 \\
&\cdot K_6 + \mathbf{AQ} \cdot \mathbf{Ap} \cdot A_{HG} \cdot K_6) \cdot (A_2 \cdot K_7 \cdot K_{51} + A_1 \cdot K_{32} \cdot K_{52} \cdot K_6 \cdot K_{62}) \cdot C_D'] \\
&\cdot \Pr\ [\mathbf{CCS}]\} \qquad \text{(D-26)}
\end{aligned}
$$

$$\Pr\left[W'\cdot U'\cdot \overline{\mathbf{CCS}}\cdot \overline{C_D'}\right] = \Pr\left\{A_2\cdot K_7\cdot \left[\mathbf{CCS} + (K_{51} + A_1\cdot K_3\cdot K_{32}\cdot K_{52}\cdot K_6\cdot K_{62})\cdot \overline{\mathbf{CCS}}\right]\cdot \overline{\mathbf{CCS}}\cdot \overline{C_D'}\right\}$$

$$= \Pr\left[A_2\cdot K_7\cdot (K_{51} + A_1\cdot K_3\cdot K_{32}\cdot K_{52}\cdot K_6\cdot K_{62})\cdot \overline{C_D'}\right]\cdot (1 - \Pr\left[\mathbf{CCS}\right]) \tag{D-27}$$

$$\Pr\left[W'\cdot V'\cdot \overline{\mathbf{CCS}}\cdot \overline{C_D'}\right] = \Pr\left\{K_3\cdot \left[\alpha_1\cdot \mathbf{CCS} + (\beta_1\cdot \overline{C_D'} + \gamma_1\cdot C_D')\cdot \overline{\mathbf{CCS}}\right]\cdot \overline{\mathbf{CCS}}\cdot \overline{C_D'}\right\} = \Pr(K_3\cdot \beta_1$$

$$\cdot\, \overline{C_D'}\cdot \overline{\mathbf{CCS}}) = \Pr\left[K_3\cdot A_1\cdot K_{32}\cdot (A_2\cdot K_7\cdot K_{51} + K_{52}\cdot K_6\cdot K_{62})\cdot \overline{C_D'}\right]$$

$$\cdot\, (1 - \Pr\left[\mathbf{CCS}\right]) \tag{D-28}$$

$$\Pr\left[W'\cdot U'\cdot V'\cdot (\mathbf{CCS} + C_D')\right] = \Pr\left\{A_2\cdot K_3\cdot K_7\cdot \left[\alpha_2\cdot \mathbf{CCS} + (\beta_2\cdot \overline{C_D'} + \gamma_2\cdot C_D')\cdot \overline{\mathbf{CCS}}\right]\cdot (\mathbf{CCS} + C_D')\right\}$$

$$= \Pr\left\{A_2\cdot K_3\cdot K_7\cdot \left[\alpha_2\cdot \mathbf{CCS} + \gamma_2\cdot C_D'\cdot \overline{\mathbf{CCS}}\right]\right\} = \Pr\left[A_2\cdot K_3\cdot K_7\right]$$

$$\cdot \left\{\Pr\left[A_1 + \mathbf{AQ}\cdot \mathbf{Ap}\cdot A_{HG}\cdot K_6\right)\cdot \Pr\left[\mathbf{CCS}\right] + \Pr\left[(A_1 + \mathbf{AQ}\cdot \mathbf{Ap}\right.\right.$$

$$\cdot\, A_{HG}\cdot K_6)\cdot (K_{51} + A_1\cdot K_{32}\cdot K_{52}\cdot K_6\cdot K_{62})\big]\cdot \Pr\left[C_D'\right]$$

$$- \Pr\left[(A_1 + \mathbf{AQ}\cdot \mathbf{Ap}\cdot A_{HG}\cdot K_6)\cdot (K_{51} + A_1\cdot K_{32}\cdot K_{52}\cdot K_6\cdot K_{62})\right]$$

$$\left.\cdot \Pr\left[C_D'\right]\cdot \Pr\left[\mathbf{CCS}\right]\right\} \tag{D-29}$$

$$\Pr\left[W'\cdot U'\cdot V'\cdot \overline{\mathbf{CCS}}\cdot \overline{C_D'}\right] = \Pr\left\{A_2\cdot K_3\cdot K_7\cdot \left[\alpha_2\cdot \mathbf{CCS} + (\beta_2\cdot \overline{C_D'} + \gamma_2\cdot C_D')\cdot \overline{\mathbf{CCS}}\right]\cdot \overline{\mathbf{CCS}}\cdot \overline{C_D'}\right\}$$

$$= \Pr\left\{A_2\cdot K_3\cdot K_7\cdot \beta_2\cdot \overline{C_D'}\cdot \overline{\mathbf{CCS}}\right\} = \Pr\left[A_2\cdot K_3\cdot K_7\cdot A_1\cdot K_{32}\cdot (K_{51} + K_{52}\right.$$

$$\left.\cdot\, K_6\cdot K_{62})\cdot \overline{C_D'}\right]\cdot \left(1 - \Pr\left[\mathbf{CCS}\right]\right) \tag{D-30}$$

APPENDIX E. Derivation of Pr $[R_t]$ for the Range-Tracking Function

The success event for the range-tracking function is

$$R_t = (\mathbf{PO})\cdot(\mathbf{AC})\cdot(\mathbf{RG})\cdot P_4\cdot K_2\cdot R_1\cdot(S_2+S_{21})\cdot(\mathrm{T})\cdot\{\mathrm{V}\cdot[(\mathbf{CCS}+C_D')\cdot(M+M_{11})+\overline{\mathbf{CCS}}\cdot\overline{C_D'}\cdot M_{21}]$$

$$+K_1\cdot\mathrm{U}\cdot[(\mathbf{CCS}+C_D')\cdot(M+M_{12})+\overline{\mathbf{CCS}}\cdot\overline{C_D'}\cdot M_{22}]\} \quad \text{(E-1)}$$

where

$$\mathrm{T} = \{C_D'\cdot(G_7+G_8)\cdot(G_9+G_{10})+\overline{C_D'}\cdot[(S_3\cdot K_4)_1\cdot G_7+(S_3\cdot K_4)_2\cdot G_8]\cdot[S_{41}\cdot G_9+S_{42}\cdot G_{10}]\}$$

$$\cdot G_9'\cdot G_{10}'\cdot K_4\cdot P_7\cdot S_3\cdot S_4 \quad \text{(E-2)}$$

$$\mathrm{U} = A_2\cdot K_5\cdot K_7\cdot[(\mathbf{CCS}+C_D')+(A_1\cdot K_{32}\cdot K_{52}\cdot K_{62}\cdot K_3\cdot K_6+K_{51})\cdot\overline{\mathbf{CCS}}\cdot\overline{C_D'}] \quad \text{(E-3)}$$

$$\mathrm{V} = K_3\cdot K_5\cdot[(A_1\cdot A_2\cdot K_7+A_1\cdot K_6+\mathbf{AQ}\cdot\mathbf{Ap}\cdot A_{HG}\cdot K_6)\cdot(\mathbf{CCS}+C_D')+A_1\cdot K_{32}\cdot(K_{51}+K_{52}\cdot K_6\cdot K_{62})$$

$$\cdot\overline{\mathbf{CCS}}\cdot\overline{C_D'}] \quad \text{(E-4)}$$

$$\mathrm{U}\cdot\mathrm{V} = A_2\cdot K_3\cdot K_5\cdot K_7\cdot[(A_1+\mathbf{AQ}\cdot\mathbf{Ap}\cdot A_{HG}\cdot K_6)\cdot(\mathbf{CCS}+C_D')+A_1\cdot K_{32}\cdot(K_{51}+K_{52}\cdot K_6\cdot K_{62})$$

$$\cdot\overline{\mathbf{CCS}}\cdot\overline{C_D'}] \quad \text{(E-5)}$$

The associated probability is

$$\Pr[R_t] = \Pr[(\mathbf{PO})\cdot(\mathbf{AC})]\cdot\Pr[\mathbf{RG}]\cdot\Pr[K_2\cdot K_4\cdot K_5\cdot P_4\cdot P_7\cdot(S_2+S_{21})\cdot S_3\cdot S_4\cdot R_1]\cdot\Pr[G_9'\cdot G_{10}']$$

$$\cdot\Pr\Big[\{C_D'\cdot(G_7+G_8)\cdot(G_9+G_{10})+\overline{C_D'}\cdot[(S_3\cdot K_4)_1\cdot G_7+(S_3\cdot K_4)_2\cdot G_8]\cdot[S_{41}\cdot G_9+S_{42}\cdot G_{10}]\}$$

$$\cdot\{K_3[(A_1\cdot A_2\cdot K_7+A_1\cdot K_6+\mathbf{AQ}\cdot\mathbf{Ap}\cdot A_{HG}\cdot K_6)\cdot(\mathbf{CCS}+C_D')+A_1\cdot K_{32}\cdot(K_{51}+K_{52}\cdot K_6\cdot K_{62})$$

$$\cdot\overline{\mathbf{CCS}}\cdot\overline{C_D'}]\cdot[(\mathbf{CCS}+C_D')\cdot(M+M_{11})+\overline{\mathbf{CCS}}\cdot\overline{C_D'}\cdot M_{21}]+K_1\cdot A_2\cdot K_7\cdot[(\mathbf{CCS}+C_D')+(A_1\cdot K_{32}$$

$$\cdot K_{52}\cdot K_{62}\cdot K_3\cdot K_6+K_{51})\cdot\overline{\mathbf{CCS}}\cdot\overline{C_D'}]\cdot[(\mathbf{CCS}+C_D')\cdot(M+M_{12})+\overline{\mathbf{CCS}}\cdot\overline{C_D'}\cdot M_{22}]\}\Big] \quad \text{(E-6)}$$

$$\begin{aligned}\Pr[R_t] = {} & \Pr[(\mathbf{PO})\cdot(\mathbf{AC})]\cdot\Pr[\mathbf{RG}]\cdot\Pr[K_2\cdot K_4\cdot P_4\cdot P_7\cdot(S_2+S_{21})\cdot S_3\cdot S_4\cdot R_1]\cdot\Pr[G_9'\cdot G_{10}'] \\ & \cdot\Big\{\Pr\Big[(T')\cdot\{\mathrm{V}\cdot[(\mathbf{CCS}+C_D')\cdot(M+M_{11})+\overline{\mathbf{CCS}}\cdot\overline{C_D'}\cdot M_{21}]\}\Big]+\Pr\Big[(T')\cdot\{K_1\cdot\mathrm{U} \\ & \cdot[(\mathbf{CCS}+C_D')\cdot(M+M_{12})+\overline{\mathbf{CCS}}\cdot\overline{C_D'}\cdot M_{22}]\}\Big]-\Pr\Big[(T')\cdot\{K_1\cdot\mathrm{U}\cdot\mathrm{V}\cdot(\mathbf{CCS}+C_D')\cdot M\}\Big]\Big\}\end{aligned} \tag{E-7}$$

$$\begin{aligned}\Pr[R_t] = {} & \Pr[(\mathbf{PO})\cdot(\mathbf{AC})]\cdot\Pr[\mathbf{RG}]\cdot\Pr[K_2\cdot K_4\cdot P_4\cdot P_7\cdot(S_2+S_{21})\cdot S_3\cdot S_4\cdot R_1]\cdot\Pr[G_9'\cdot G_{10}'] \\ & \cdot\Big\{\Pr\{(\mathrm{T}')\cdot\mathrm{V}\cdot[(\mathbf{CCS}+C_D')\cdot(M+M_{11})]\}+\Pr\{(\mathrm{T}')\cdot\mathrm{V}\cdot\overline{\mathbf{CCS}}\cdot\overline{C_D'}\cdot M_{21}\}+\Pr\{(\mathrm{T}')\cdot K_1\cdot\mathrm{U} \\ & \cdot[(\mathbf{CCS}+C_D')\cdot(M+M_{12})]\}+\Pr\{(\mathrm{T}')\cdot K_1\cdot\mathrm{U}\cdot\overline{\mathbf{CCS}}\cdot\overline{C_D'}\cdot M_{22}\}-\Pr\{(\mathrm{T}')\cdot[K_1\cdot\mathrm{U}\cdot\mathrm{V}\cdot(\mathbf{CCS} \\ & +C_D')\cdot M]\}\Big\}\end{aligned} \tag{E-8}$$

$$\begin{aligned}\Pr[R_t] = {} & \Pr[(\mathbf{PO})\cdot(\mathbf{AC})]\cdot\Pr[\mathbf{RG}]\cdot\Pr[K_2\cdot K_4\cdot P_4\cdot P_7\cdot(S_2+S_{21})\cdot S_3\cdot S_4\cdot R_1]\cdot\Pr[G_9'\cdot G_{10}'] \\ & \cdot\{\Pr[(M+M_{11})\cdot K_3\cdot K_5\cdot(A_1\cdot A_2\cdot K_7+A_1\cdot K_6+\mathbf{AQ}\cdot\mathbf{Ap}\cdot A_{HG}\cdot K_6)]\cdot\Pr[(\mathbf{CCS}+C_D')\cdot(\mathrm{T}')] \\ & +\Pr[M_{21}\cdot A_1\cdot K_3\cdot K_5\cdot K_{32}(K_{51}+K_{52}\cdot K_6\cdot K_{62})]\cdot\Pr[\overline{\mathbf{CCS}}\cdot\overline{C_D'}\cdot(\mathrm{T}')]+\Pr[(M+M_{12}) \\ & \cdot K_1\cdot K_5\cdot K_7\cdot A_2]\cdot\Pr[(\mathbf{CCS}+C_D')\cdot(\mathrm{T}')]+\Pr[M_{22}\cdot K_1\cdot A_2\cdot K_5\cdot K_7\cdot(A_1\cdot K_{32}\cdot K_{62}\cdot K_3\cdot K_6 \\ & +K_{51})]\cdot\Pr[\overline{\mathbf{CCS}}\cdot\overline{C_D'}\cdot(\mathrm{T}')]-\Pr[M\cdot K_1\cdot A_2\cdot K_3\cdot K_5\cdot K_7\cdot(A_1+\mathbf{AQ}\cdot\mathbf{Ap}\cdot A_{HG}\cdot K_6) \\ & \cdot\Pr[(\mathbf{CCS}+C_D')\cdot(\mathrm{T}')]\}\end{aligned} \tag{E-9}$$

Since from Eq. (B-9)

$$\mathrm{T}' = F_1\cdot C_D' + F_2\cdot\overline{C_D'}$$

one obtains

$$\Pr[(\mathbf{CCS}+C_D')\cdot(\mathrm{T}')] = \Pr[F_1\cdot C_D' + F_2\cdot\overline{C_D'}\cdot\mathbf{CCS}] \tag{E-10}$$

and

$$\Pr\,[(\overline{\mathbf{CCS}}\cdot\overline{C_D'})\cdot(\mathrm{T}')] = \Pr\,[F_2\cdot\overline{C_D'}\cdot\overline{\mathbf{CCS}}] \tag{E-11}$$

Therefore,

$$\Pr\,[R_t] = \mu_0\cdot\{\mu_1\cdot\Pr\,[(\mathbf{CCS}+C_D')\cdot(\mathrm{T}')] + \mu_2\cdot\Pr\,[\overline{\mathbf{CCS}}\cdot\overline{C_D'}\cdot(\mathrm{T}')]\}$$

where

$$\mu_0 = \Pr\,[(\mathbf{PO}\cdot(\mathbf{AC})]\cdot\Pr\,[\mathbf{RG}]\cdot\Pr\,[K_2\cdot K_4\cdot P_4\cdot P_7\cdot(S_2+S_{21})\cdot S_3\cdot S_4\cdot R_1]\cdot\Pr\,[G_9'\cdot G_{10}'] \tag{E-12}$$

$$\begin{aligned}\mu_1 = {}& \Pr\,[(M+M_{11})\cdot K_3\cdot K_5\cdot(A_1\cdot A_2\cdot K_7 + A_1\cdot K_6 + \mathbf{AQ}\cdot\mathbf{Ap}\cdot A_{HG}\cdot K_6)] + \Pr\,[(M+M_{12})\cdot K_1\cdot K_5\cdot K_7\cdot A_2]\\ & - \Pr\,[M\cdot K_1\cdot A_2\cdot K_3\cdot K_5\cdot K_7\cdot(A_1 + \mathbf{AQ}\cdot\mathbf{Ap}\cdot A_{HG}\cdot K_6)]\end{aligned} \tag{E-13}$$

$$\begin{aligned}\mu_2 = {}& \Pr\,[M_{21}\cdot A_1\cdot K_3\cdot K_5\cdot K_{32}\cdot(K_{51}+K_{52}\cdot K_6\cdot K_{62})] + \Pr\,[M_{22}\cdot K_1\cdot A_2\cdot K_5\cdot K_7\cdot(A_1\cdot K_{32}\cdot K_{52}\cdot K_{62}\\ & \cdot K_3\cdot K_6 + K_5)]\end{aligned} \tag{E-14}$$

APPENDIX F. Derivation of Pr[(CCS) $\cdot D_O$ + (CCS + C_D') $\cdot D_T$ for the Telemetry Function

The required probability is

$$\Pr\ [(\mathbf{CCS})\cdot D_O + (\mathbf{CCS} + C_D')\cdot D_T] = \Pr\ [(\mathbf{CCS})\cdot D_O] + \Pr\ [(\mathbf{CCS} + C_D')\cdot D_T] - \Pr\ [(\mathbf{CCS})\cdot D_O \cdot D_T] \quad \text{(F-1)}$$

Since

$$\begin{aligned} D_O = {} & (\mathbf{PO})\cdot(\mathbf{AC})\cdot K_4\cdot K_5\cdot P_4\cdot P_7\cdot S_3\cdot S_4\cdot [G_7\cdot G_{11}\cdot (S_3\cdot K_4)_1 + G_8\cdot G_{12}\cdot (S_3\cdot K_4)_2]\cdot [G_9\cdot S_{41} + G_{10}\cdot S_{42}] \\ & \cdot G_9'\cdot G_{10}'\cdot [(A_2\cdot K_7 + A_1\cdot K_3\cdot K_6 + \mathbf{AQ}\cdot \mathbf{Ap}\cdot A_{HG}\cdot K_6)\cdot \mathbf{CCS} + (A_2\cdot K_7\cdot K_{51} + A_1\cdot K_3\cdot K_{32}\cdot K_{52} \\ & \cdot K_6\cdot K_{62})\cdot \overline{\mathbf{CCS}}] \end{aligned} \quad \text{(F-2)}$$

then

$$\begin{aligned} (\mathbf{CCS})\cdot D_O = {} & (\mathbf{PO})\cdot(\mathbf{AC})\cdot K_4\cdot K_5\cdot P_4\cdot P_7\cdot S_3\cdot S_4\cdot [G_7\cdot G_{11}\cdot (S_3\cdot K_4)_1 + G_8\cdot G_{12}\cdot (S_3\cdot K_4)_2] \\ & \cdot [G_9\cdot S_{41} + G_{10}\cdot S_{42}]\cdot G_9'\cdot G_{10}'\cdot [A_2\cdot K_7 + A_1\cdot K_3\cdot K_6 + \mathbf{AQ}\cdot \mathbf{Ap}\cdot A_{HG}\cdot K_6]\cdot \mathbf{CCS} \end{aligned} \quad \text{(F-3)}$$

The associated probability is

$$\begin{aligned} \Pr\ [(\mathbf{CCS})\cdot D_O] = {} & \Pr\ [(\mathbf{PO})\cdot(\mathbf{AC})]\cdot \Pr\ [K_4\cdot K_5\cdot P_4\cdot P_7\cdot S_3\cdot S_4\cdot G_9'\cdot G_{10}']\cdot \Pr\ \{[G_7\cdot G_{11}\cdot (S_3\cdot K_4)_1 \\ & + G_8\cdot G_{12}\cdot (S_3\cdot K_4)_2]\cdot [G_9\cdot S_{41} + G_{10}\cdot S_{42}]\}\cdot \Pr\ (A_2\cdot K_7 + A_1\cdot K_3\cdot K_6 + \mathbf{AQ}\cdot \mathbf{Ap} \\ & \cdot A_{HG}\cdot K_6)\cdot \Pr\ (\mathbf{CCS}) = \Pr\ [(\mathbf{PO})\cdot(\mathbf{AC})]\cdot \Pr\ (K_4\cdot K_5\cdot P_4\cdot P_7\cdot S_3\cdot S_4\cdot G_9'\cdot G_{10}') \\ & \cdot \Pr\ \{[G_7\cdot G_{11} (S_3\cdot K_4)_1 + G_8\cdot G_{12}\cdot (S_3\cdot K_4)_2]\cdot [G_9\cdot S_{41} + G_{10}\cdot S_{42}]\}\cdot \{\Pr\ [A_2\cdot K_7] \\ & + \Pr\ [A_1\cdot K_3\cdot K_6] + \Pr\ [\mathbf{AQ}\cdot \mathbf{Ap}\cdot A_{HG}\cdot K_6] - \Pr\ [A_1\cdot A_2\cdot K_3\cdot K_6\cdot K_7] - \Pr\ [\mathbf{AQ}\cdot \mathbf{Ap}\cdot A_{HG} \\ & \cdot A_2\cdot K_6\cdot K_7] - \Pr\ [\mathbf{AQ}\cdot \mathbf{Ap}\cdot A_{HG}\cdot A_1\cdot K_3\cdot K_6] + \Pr\ [\mathbf{AQ}\cdot \mathbf{Ap}\cdot A_{HG}\cdot A_1\cdot A_2\cdot K_3 \\ & \cdot K_6\cdot K_7]\}\cdot \Pr\ [\mathbf{CCS}] \end{aligned} \quad \text{(F-4)}$$

Since

$$D_T = (\mathbf{PO})\cdot(\mathbf{AC})\cdot K_4\cdot P_4\cdot P_7\cdot S_3\cdot S_4\cdot\{[C'_D + \overline{C'_D}\cdot(S_3\cdot K_4)_1]\cdot G_7 + [C'_D + \overline{C'_D}\cdot(S_3\cdot K_4)_2]\cdot G_8\}$$

$$\cdot\{[C'_D + \overline{C'_D}\cdot S_{41}]\cdot G_9 + [C'_D + \overline{C'_D}\cdot S_{42}]\cdot G_{10}\}\cdot G'_9\cdot G'_{10}\cdot\{[R_2\cdot K_1\cdot(S_2 + S_{22})\cdot \mathrm{U} + R_1\cdot K_2$$

$$\cdot(S_2 + S_{21})\cdot \mathrm{V}]\cdot[(\mathbf{CCS} + C'_D)\cdot(M + M_{11}) + \overline{\mathbf{CCS}}\cdot\overline{C'_D}\cdot M_{21}] + [R_1\cdot K_1\cdot K_2\cdot(S_2 + S_{21})\cdot \mathrm{U}$$

$$+ R_2\cdot K_1\cdot K_2\cdot(S_2 + S_{22})\cdot \mathrm{V}]\cdot[(\mathbf{CCS} + C'_D)\cdot(M + M_{12}) + \overline{\mathbf{CCS}}\cdot\overline{C'_D}\cdot M_{22}]\} \tag{F-5}$$

then

$$(\mathbf{CCS} + C'_D)\cdot D_T = (\mathbf{PO})\cdot(\mathbf{AC})\cdot K_4\cdot P_4\cdot P_7\cdot S_3\cdot S_4\cdot\{[C'_D + C'_D\cdot(S_3\cdot K_4)_1]\cdot G_7 + [C'_D + \overline{C'_D}\cdot(S_3\cdot K_4)_2]\cdot G_8\}$$

$$\cdot\{[C'_D + \overline{C'_D}\cdot S_{41}]\cdot G_9 + [C'_D + \overline{C'_D}\cdot S_{42}]\cdot G_{10}\}\cdot G'_9\cdot G'_{10}\cdot\{[R_2\cdot K_1\cdot(S_2 + S_{22})\cdot \mathrm{U}''$$

$$+ R_1\cdot K_2\cdot(S_2 + S_{21})\cdot \mathrm{V}'']\cdot(M + M_{11}) + [R_1\cdot K_1\cdot K_2\cdot(S_2 + S_{21})\cdot \mathrm{U}'' + R_2\cdot K_1\cdot K_2$$

$$\cdot(S_2 + S_{22})\cdot \mathrm{V}'']\cdot(M + M_{12})\}\cdot(\mathbf{CCS} + C'_D) \tag{F-6}$$

where

$$\mathrm{U}'' = A_2\cdot K_5\cdot K_7 \tag{F-7}$$

$$\mathrm{V}'' = K_3\cdot K_5\cdot(A_1\cdot A_2\cdot K_7 + A_1\cdot K_6 + \mathbf{AQ}\cdot\mathbf{Ap}\cdot A_{HG}\cdot K_6) \tag{F-8}$$

$$\mathrm{U}''\cdot \mathrm{V}'' = A_2\cdot K_3\cdot K_5\cdot K_7\cdot(A_1 + \mathbf{AQ}\cdot\mathbf{Ap}\cdot A_{HG}\cdot K_6) \tag{F-9}$$

The associated probability is

$$\Pr\,[(\mathbf{CCS} + C'_D)\cdot D_T] = \Pr\,[(\mathbf{PO})\cdot(\mathbf{AC})]\cdot\Pr\,[K_4\cdot P_4\cdot P_7\cdot S_3\cdot S_4\cdot G'_9\cdot G'_{10}]\cdot\Pr\,\{[R_2\cdot K_1\cdot(S_2 + S_{22})\cdot \mathrm{U}''$$

$$+ R_1\cdot K_2\cdot(S_2 + S_{21})\cdot \mathrm{V}'']\cdot(M + M_{11}) + [R_1\cdot K_1\cdot K_2\cdot(S_2 + S_{21})\cdot \mathrm{U}'' + R_2\cdot K_1\cdot K_2$$

$$\cdot(S_2 + S_{22})\cdot \mathrm{V}'']\cdot(M + M_{12})\}\cdot\Pr\Big[\{C'_D\cdot(G_7 + G_8)\cdot(G_9 + G_{10}) + \overline{C'_D}\cdot[(S_3\cdot K_4)_1$$

$$\cdot G_7 + (S_3\cdot K_4)_2\cdot G_8]\cdot[S_{41}\cdot G_9 + S_{42}\cdot G_{10}]\}\cdot(\mathbf{CCS} + C'_D)\Big] \tag{F-10}$$

The only remaining term to be evaluated is

$$(\mathbf{CCS})\cdot D_O\cdot D_T = (\mathbf{PO})\cdot(\mathbf{AC})\cdot K_4\cdot K_5\cdot P_4\cdot P_7\cdot S_3\cdot S_4\cdot G_9'\cdot G_{10}'\cdot [G_7\cdot G_{11}\cdot(S_3\cdot K_4)_1 + G_8\cdot G_{12}\cdot(S_3\cdot K_4)_2]$$
$$\bullet\ [G_9\cdot S_{41} + G_{10}\cdot S_{42}]\cdot\Big[[A_2\cdot K_7 + A_1\cdot K_3\cdot K_6 + \mathbf{AQ}\cdot\mathbf{Ap}\cdot A_{HG}\cdot K_6]\cdot\{[R_2\cdot K_1$$
$$\bullet\ (S_2 + S_{22})\cdot \mathrm{U}'' + R_1\cdot K_2\cdot(S_2 + S_{21})\cdot \mathrm{V}'']\cdot(M + M_{11}) + [R_1\cdot K_1\cdot K_2\cdot(S_2 + S_{21})\cdot \mathrm{U}''$$
$$+ R_2\cdot K_1\cdot K_2\cdot(S_2 + S_{22})\cdot \mathrm{V}'']\cdot(M + M_{12})\}\Big]\cdot\mathbf{CCS} \qquad \text{(F-11)}$$

for which the associated probability is

$$\Pr[(\mathbf{CCS})\cdot D_O\cdot D_T] = \Pr[(\mathbf{PO})\cdot(\mathbf{AC})]\cdot\Pr[K_4\cdot K_5\cdot P_4\cdot P_7\cdot S_3\cdot S_4\cdot G_9'\cdot G_{10}']\cdot\Pr\{[G_7\cdot G_{11}\cdot(S_3\cdot K_4)_1$$
$$+ G_8\cdot G_{12}\cdot(S_3\cdot K_4)_2]\cdot[G_9\cdot S_{41} + G_{10}\cdot S_{42}]\}\cdot\Pr\Big[[A_2\cdot K_7 + A_1\cdot K_3\cdot K_6$$
$$+ \mathbf{AQ}\cdot\mathbf{Ap}\cdot A_{HG}\cdot K_6]\cdot\{[R_2\cdot K_1\cdot(S_2 + S_{22})\cdot \mathrm{U}'' + R_1\cdot K_2\cdot(S_2 + S_{21})\cdot \mathrm{V}'']$$
$$\bullet\ (M + M_{11}) + [R_1\cdot K_1\cdot K_2\cdot(S_2 + S_{21})\cdot \mathrm{U}'' + R_2\cdot K_1\cdot K_2\cdot(S_2 + S_{22})\cdot \mathrm{V}'']$$
$$\bullet\ (M + M_{12})\}\Big]\cdot\Pr[\mathbf{CCS}] \qquad \text{(F-12)}$$